§1.5 Complex Numbers

Complex number: a number of the form a + bi, where a and b are real numbers. a is called the REAL part of the complex number a + bi, bi is called the IMAGINARY part of the complex number a + bi.

Imaginary number: a complex number of the form a + bi, where b is nonzero.

Standard Form of a complex number: a + bi or a + ib
(Discuss $i\sqrt{5}$ & $\sqrt{5}i$)

Definition of i: $i = \sqrt{-1}$ or $i^2 = -1$

Definition of $\sqrt{-a}$ If $a > 0$, then $\sqrt{-a} = i\sqrt{a}$ Example: $\sqrt{-16}$

Caution: When working with negative radicands, be sure to use the definition $\sqrt{-a} = i\sqrt{a}$ before using any of the other rules for radicals.

OPERATIONS WITH COMPLEX NUMBERS

Addition or Subtraction of Complex Numbers:

1. Combine the real parts.
2. Combine the imaginary parts.
3. Leave the result in the form a + bi.

Note: Add (or subtract) the real numbers then add the imaginary numbers.

example 1: a) $(-9 + 7i) + (3 - 15i)$ b) $(12 - 5i) - (8 - 3i)$

Multiplication of Complex Numbers:

1. Multiply the numbers as if they are two binomials (FOIL METHOD).
2. Substitute -1 for i^2
3. Combine the like terms and leave the result in the form a + bi.

example 2: a) $(2 - 3i)(3 + 4i)$ b) $(4 + 3i)^2$

Properties of Complex Conjugates: for real numbers a and b:

$$(a + bi)(a - bi) = a^2 + b^2$$

Division of Complex Numbers:

1. Write the division as a fraction.
2. Multiply the numerator and denominator by the conjugate of the denominator:

$$\frac{a+bi}{c+di}\cdot\frac{c-di}{c-di}.$$

3. Multiply and simplify in the numerator (by FOIL). Multiply and simplify in the denominator to a real number (by FOIL).
4. Write the result in the form a + bi.

example 3: a) $\dfrac{3+2i}{5-i}$

Powers of i: $i^1 = i$ $i^2 = -1$ $i^3 = -i$ $i^4 = 1$

example 4: a) i^{13} b) i^{56}