

§3.1 Quadratic Functions

Polynomials are continuous (no breaks in the graph) and smooth (no sharp angles, only rounded curves)

Quadratic Function:

A function f is a quadratic function if

$$f(x) = ax^2 + bx + c,$$

where a , b and c are real numbers and $a \neq 0$.

The graph of a quadratic function is a **parabola**.

The line of symmetry of a parabola is called the **axis**.

The highest or lowest point on a parabola is called the **vertex**. (Note: this depends on the sign of the coefficient of x^2)

A vertical or horizontal shift of the standard parabola $y = x^2$ is called a **translation**.

(hint: note where vertex is!)

Graph of a Quadratic function (parabola):

The quadratic function defined by $f(x) = ax^2 + bx + c$, can be written in the form:

$$y = f(x) = a(x - h)^2 + k, \quad a \neq 0 \quad \text{where } h = -\frac{b}{2a} \text{ and } k = f(h).$$

The graph of f has the following characteristics:

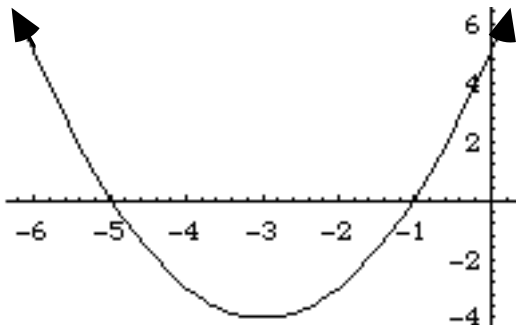
- 1) It is a parabola with vertex (h, k) , and the vertical line $x = h$ as axis.
- 2) It opens upward if $a > 0$ and downward if $a < 0$.
- 3) It is broader than $y = x^2$ if $0 < |a| < 1$ and narrower than $y = x^2$ if $|a| > 1$.
- 4) The y - intercept is $f(0) = c$. (where c is some real number)
- 5) The x - intercepts are found by solving $ax^2 + bx + c = 0$

Examples:

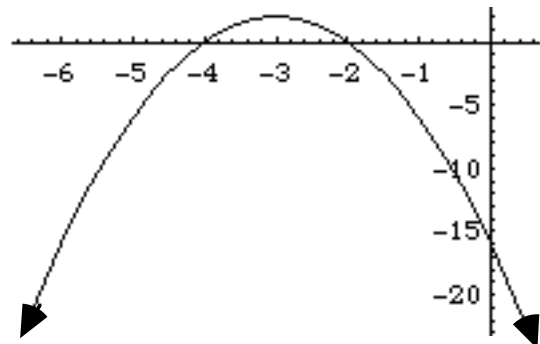
1) $y = (x - (-3))^2 - 4$
(graph form)

2) $y = -2x^2 - 12x - 16$
(standard form)

X - int. : $(-5, 0), (-1, 0)$
y - int.: $(0, 5)$
vertex: $(-3, -4)$ axis: $x = -3$
D: $(-\infty, \infty)$ R: $[-4, \infty)$



X - int.: $(-4, 0), (-2, 0)$
y - int.: $(0, -16)$
vertex: $(-3, 2)$ axis: $x = -3$
D: $(-\infty, \infty)$ R: $(-\infty, 2]$



Example 1: Find the vertex (h, k) for the following quadratic equation: $y = x^2 + 6x + 5$.

Example 2: Graph the parabola, $y = 2x^2 - 4x + 5$, give the vertex, axis, domain, and range, x and y intercepts.

Example 3: Write the parabola, $y = x^2 - 2x + 3$ in "graphing form".