§3.3 Polynomial and Synthetic Division

Division Algorithm :

For any polynomial P(x) and any complex number k , there exists a unique polynomial Q(x) and number r such that: $$P(x) = d(x) * Q(x) + r.$$

Example 1: Divide

a) $6q^3 - 17q^2 + 22q - 23$ by $2q – 3$

b) $3x^3 - 2x^2 - 150$ by $x – 4$

Synthetic Division :

$$
\begin{array}{rl}
 & \phantom{3x^3-{}}3x^2+10x+40 \\
x-4 \,\big) & \overline{3x^3-2x^2+0x-150} \\
(-) & \underline{3x^3-12x^2} \\
 & \qquad 10x^2+0x \\
(-) & \qquad \underline{10x^2-40x} \\
 & \qquad\qquad 40x-150 \\
(-) & \qquad\qquad \underline{40x-160} \\
 & \qquad\qquad\qquad 10
\end{array}
$$

Answer : $3x^2+10x+40+\dfrac{10}{x-4}$

$$
\begin{array}{r|rrrr}
4 & 3 & -2 & 0 & -150 \\
 & & 12 & 40 & 160 \\
\hline
 & 3 & 10 & 40 & 10
\end{array}
$$

Answer : $3x^2+10x+40+\dfrac{10}{x-4}$

Example 2: Divide by synthetic division. x^4-10x^2-2x+4 by $x+3$

The Remainder Theorem

If a polynomial f(x) is divided by x – k, the remainder is equal to f(k).

Example 3:
Use the remainder theorem and synthetic division to find f(–2)
for $f(x) = 5x^3 - 6x^2 - 28x + 8.$

The Factor Theorem

The polynomial x – k is a factor of the polynomial f(x) if and only if f(k) = 0.

Example 4:
Decide whether the second polynomial is a factor of the first.

a) $f(x) = 4x^3 + 24x^2 + 48x + 32;$ x + 2

b) $f(x) = 2x^4 + 3x^2 - 5x + 7;$ x – 1