§3.4 Zeros of Polynomial Functions

Rational Zero Test:

If the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$
has integer coefficients, every rational zero of f(x) has the form

$$\text{Rational Zero} = \frac{p}{q}$$

where p and q have no common factors other than 1, and

p is a factor of the constant term a_0 and
q is a factor of the leading coefficient a_n

$$\text{Possible rational zeros} = \frac{\text{factors of constant term } a_0}{\text{factors of leading coefficient } a_n}$$

Example 1: List the possible rational zeros for each function

a) $f(x) = x^4 - x^3 + x^2 - 3x - 6$

b) $f(x) = 2x^3 + 3x^2 - 8x + 3$

Now that we have a list of possible zeros, we need to use synthetic division to determine which possible zeros are actual zeros.

Example 2: Use synthetic division to find <u>all</u> the zeros for the function.

$f(x) = 2x^3 + 3x^2 - 8x + 3$

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Example 3: List the zeros for each function.

a) $f(x) = x - 2$

b) $f(x) = x^2 - 6x + 9$

c) $f(x) = x^3 + 4x$

d) $f(x) = x^4 - 1$

Example 4: Write $f(x) = x^5 + x^3 + 2x^2 - 12x + 8$ as the product of linear factors, and list all of its zeros.