

## Guidelines for Graphing Rational Functions

example a.)  $f(x) = \frac{2(x^2 - 9)}{x^2 - 4}$

1. Find and plot the x-intercepts. (Set numerator = 0 and solve for x).

$$2(x^2 - 9) = 0$$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

2. Find and plot the y-intercepts. (Let  $x = 0$  and solve for  $y$ )

$$f(0) = \frac{2(0^2 - 9)}{0^2 - 4} = \frac{9}{2}$$

3. Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$$x^2 - 4$$

$$x^2 = 4$$

$$x = \pm 2$$

4. Find and plot the Horizontal Asymptotes. (Top heavy, Bottom heavy or Same)

$$f(x) = \frac{2(x^2 - 9)}{x^2 - 4} \quad \text{Rule 2} \quad \text{Numerator and denominator have the same degree.}$$

$y = 2$  H.A.

5. Find and plot the Slant Asymptotes. (Divide numerator by denominator.)

None ! Only have these if Numerator is exactly 1 degree higher than denominator!

6. Find where the graph will intersect its nonvertical asymptote by solving  $f(x) = k$ , where  $k$  is the  $y$ -value of the horizontal asymptote, or  $f(x) = mx + b$ , where  $y = mx + b$  is the equation of the oblique asymptote.

$$\text{Solve } 2 = \frac{2(x^2 - 9)}{x^2 - 4} \quad (\text{No solution!}) \quad \text{No oblique asymptotes.}$$

7. Plot at least one point between and beyond each x-intercept and vertical asymptotes.

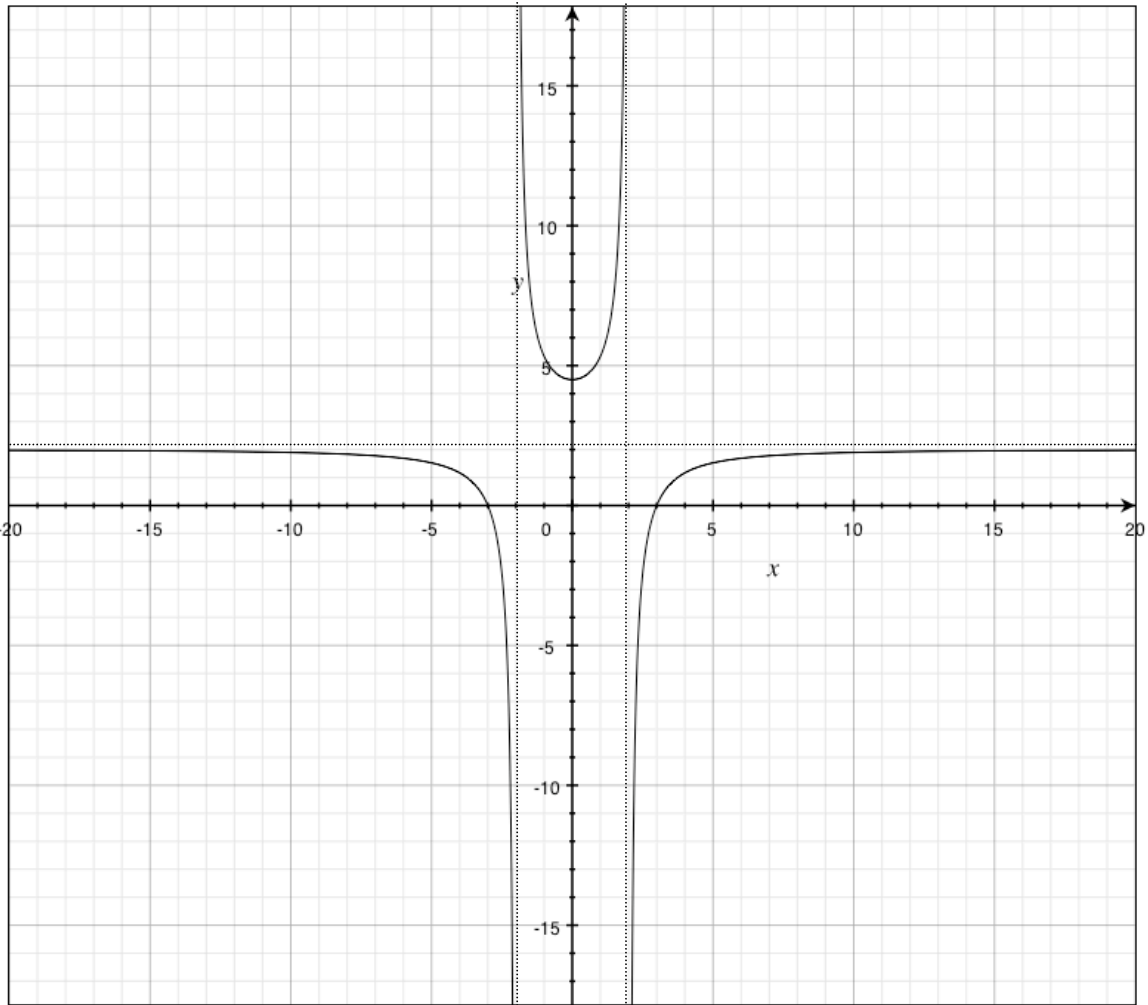
Remember Test Points ?

Choose test points carefully!

$x = -4$	$x = -2.5$	$x = 0$	$x = 2.5$	$x = 4$			$x = -1$	$x = 1$	
$y = 1.16$	$y = -2.4$	$y = 4.5$	$y = -2.4$	$y = 1.16$			$y = 5.3$	$y = 5.3$	

Note: YOU STILL MAY HAVE TO PLOT ADDITIONAL POINTS !

**Use smooth curves to complete the graph between and beyond the vertical asymptotes.**



Example Sketch the graph and provide information about intercepts and asymptotes.

$$f(x) = \frac{x}{x^2 - x - 2}$$

1. Find and plot the x-intercepts. (Set numerator = 0 and solve for x)

$$x = 0$$

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2. Find and plot the y-intercepts. (Let x = 0 and solve for y)

$$f(0) = \frac{0}{0^2 - 0 - 2} = 0$$

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3. Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$$\begin{aligned}x^2 - x - 2 &= 0 \\(x + 1)(x - 2) &= 0 \\x &= -1 \text{ and } x = 2\end{aligned}$$

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4. Find and plot the Horizontal Asymptotes. (Top heavy, Bottom heavy or Same)

(Rule 1)  $y = 0$

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5. Find and plot the Slant Asymptotes. (Divide numerator by denominator.)

None

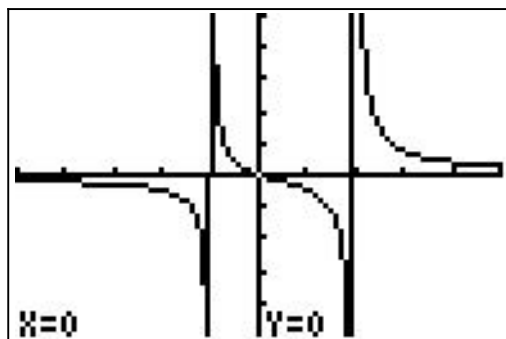
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6. Plot at least one point between and beyond each x-intercept and vertical asymptotes.

choose:

x = -2	x = -.5	x = 1	x = 3
y = -.5	y = .4	y = -.5	y = .75

Note: YOU MAY WANT TO PICK MORE POINTS TO GET A BETTER GRAPH !



ANSWER:

Example Sketch the graph and provide information about intercepts and asymptotes.

$$f(x) = \frac{x^2 - x - 2}{x - 1}$$

1. Find and plot the x-intercepts. (Set numerator = 0 and solve for x)

$$\begin{aligned} x^2 - x - 2 &= 0 \\ (x + 1)(x - 2) &= 0 \\ x &= -1 \text{ and } x = 2 \end{aligned}$$

2. Find and plot the y-intercepts. (Let x = 0 and solve for y)

$$f(0) = \frac{0^2 - 0 - 2}{0 - 1} = \frac{-2}{-1} = 2$$

3. Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$$(x - 1) = 0 \quad x = 1$$

4. Find and plot the Horizontal Asymptotes. (Top heavy, Bottom heavy or Same)

(Rule 3) Top Heavy none !

5. Find and plot the Slant Asymptotes. (Divide numerator by denominator.)

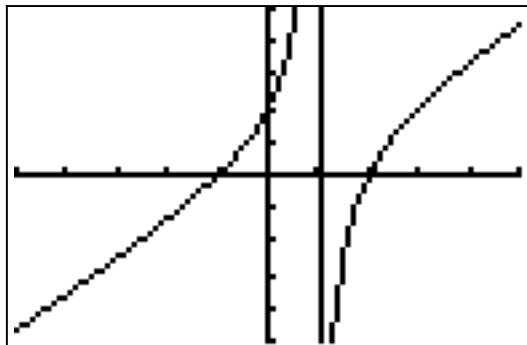
$$\begin{array}{r} x - 1 \overline{) x^2 - x - 2} \\ \underline{-x^2 + x} \phantom{- 2} \\ \phantom{x^2 - } x - 2 \end{array} \quad y = x$$

6. Plot at least one point between and beyond each x-intercept and vertical asymptotes.

choose:

x = -2	x = 0	x = 1.5	x = 3
y = -1.3	y = 2	y = -2.5	y = 2

Note: YOU MAY WANT TO PICK MORE POINTS TO GET A BETTER GRAPH !



ANSWER: