

**Directions:** To receive partial credit you must show your work on a problem.

Circle final answers. All problems are 5 points each.

For the quadratic function  $y = -2x^2 - 12x - 16$  find the following.

1. Vertex =  $\boxed{(-3, 2)}$

$$h = \frac{-b}{2a} = \frac{-(-12)}{2(-2)} = \frac{12}{-4} = -3$$

$$k = f(h) = f(-3) = -2(-3)^2 - 12(-3) - 16$$

$$= -18 + 36 - 16$$

$$= 2$$

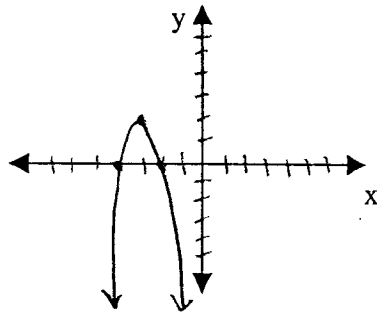
2. x and y intercepts

x-int =  $\boxed{-4, -2}$       y-int =  $\boxed{-16}$

$$\begin{array}{l|l} (y=0) & (x=0) \\ \hline \frac{0}{-2} = \frac{-2x^2 - 12x - 16}{-2} & y = -2(0)^2 - 12(0) - 16 \\ 0 = x^2 + 6x + 8 & \boxed{y = -16} \\ 0 = (x+4)(x+2) & \\ \hline \boxed{x = -4} & \boxed{x = -2} \end{array}$$

For the quadratic function  $y = -2x^2 - 12x - 16$  find the following.

3. Graph



4. Domain and Range

D =  $\boxed{(-\infty, \infty)}$       R =  $\boxed{(-\infty, 2]}$

Divide the following polynomials.

5.  $(2x^2 + 10x + 12) \div (x + 3)$

$$\begin{array}{r} -3 \overline{) 2 \ 10 \ 12} \\ \underline{-6 \ -12} \phantom{0} \\ 2 \ 4 \ 0 \end{array}$$

$\boxed{2x + 4}$

Divide the following polynomials.

6.  $(4x^3 - 7x^2 - 11x + 5) \div (4x + 5)$

$$\begin{array}{r} \boxed{x^2 - 3x + 1} \\ 4x+5 \overline{) 4x^3 - 7x^2 - 11x + 5} \\ \ominus 4x^3 \oplus 5x^2 \\ \hline -12x^2 - 11x \\ \oplus 12x^2 \oplus 15x \\ \hline 4x + 5 \\ \ominus 4x \oplus 5 \\ \hline 0 \end{array}$$

Divide the following polynomials.

7.  $(5x^3 - 6x^2 + 8) \div (x - 4)$

$$\begin{array}{r} 4 \overline{) 5 \quad -6 \quad 0 \quad 8} \\ \underline{20 \quad 56 \quad 224} \\ 5 \quad 14 \quad 56 \quad 232 \\ \underline{20 \quad 56 \quad 224} \\ 5x^2 + 14x + 56 + \frac{232}{x-4} \end{array}$$

Divide the following polynomials.

8.  $(7x + 3) \div (x + 2)$

$$\begin{array}{r} -2 \overline{) 7 \quad 3} \\ \underline{-14} \\ 7 \quad -11 \\ \boxed{7 - \frac{11}{x+2}} \end{array}$$

Use synthetic division and the factor theorem to determine whether the second polynomial is a factor of the first. State YES or NO

9.  $f(x) = x^4 - 25x^2 + 144; \quad x + 3$

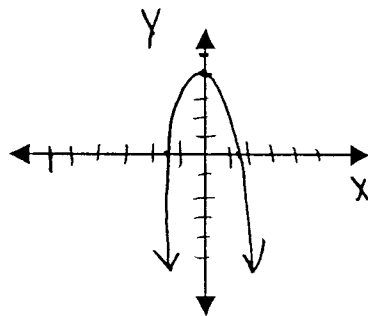
$$\begin{array}{r} -3 \overline{) 1 \quad 0 \quad -25 \quad 0 \quad 144} \\ \underline{-3 \quad 9 \quad 48 \quad -144} \\ 1 \quad -3 \quad -16 \quad 48 \quad \textcircled{0} \end{array}$$

yes, remainder is zero!

Describe the transformation that occurs in the function. Remember to find the basic function first. Also sketch the graph.

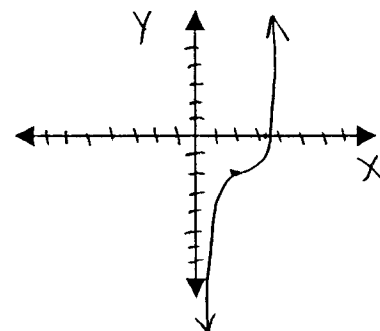
10.  $f(x) = -x^4 + 4$

Description: looks like  $y = x^4$   
reflect over x-axis  
shift up 4.



11.  $f(x) = (x - 2)^3 - 2$

Description: looks like  $y = x^3$   
slide 2 right  
and 2 down



Graph the polynomial function. **SHOW ALL** the steps we discussed in class.

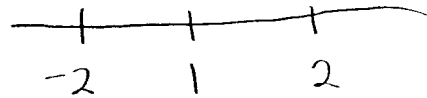
$$y = (x+2)(x-1)(x-2)(x+2)$$

12. Find the x and y intercepts.

set each = 0,

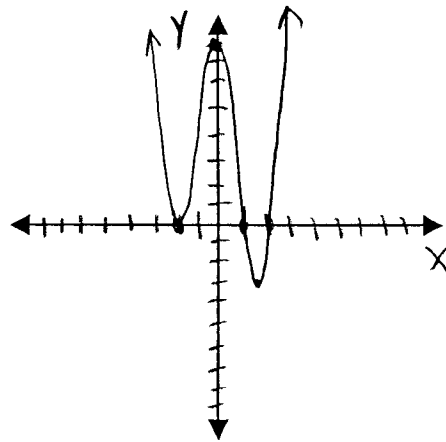
$$\begin{array}{c|c|c|c} x+2=0 & x-1=0 & x-2=0 & x+2=0 \\ \hline x=-2 & x=1 & x=2 & x=-2 \end{array}$$

13. Find the test point information for the x-intercepts.



X	Y
-3	20
0	8
1.5	-3.06
3	50

14. Use the previous work (#12 and #13) to graph the polynomial.



Find the zeros of the polynomial and state the multiplicity of each.

15.  $P(x) = x^2(x^2 - 4)(x+3)^2$

since factored  
set each = 0

$$\begin{array}{c|c|c} x^2=0 & x^2-4=0 & (x+3)^2=0 \\ \hline x=0 & x^2=4 & x+3=0 \\ \hline x=0 & x=+2 & x=-3 \\ \hline & x=-2 & x=-3 \end{array}$$

twice twice

Use the Rational Zero Theorem to list possible rational zeros for the polynomial.

16.  $P(x) = 2x^3 + x^2 - 25x + 12$

$$p = 12 \Rightarrow \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$q = 2 \Rightarrow \pm 1, \pm 2$$

$$\frac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 4, \pm 6, \pm 12$$

Use the Rational Zero Theorem to list possible rational zeros for the polynomial.

17.  $P(x) = x^5 - 32$

$p = 32 \Rightarrow \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$   
 $q = 1 \Rightarrow \pm 1$

$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$

Factor each polynomial into linear factors and/or quadratic factors that are irreducible over the reals.

18.  $P(x) = x^3 - x^2 - 2x$

factor!

$x(x^2 - x - 2)$

$x(x-2)(x+1)$

Find the zeros of the polynomial.

19.  $P(x) = 2x^4 - 17x^3 + 4x^2 + 35x - 24$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$   
 $\pm \frac{1}{2}, \pm \frac{3}{2}$

1  $\begin{array}{r|rrrrr} 2 & -17 & 4 & 35 & -24 & \\ & & 2 & -15 & -11 & 24 \\ \hline & 2 & -15 & -11 & 24 & \textcircled{0} \end{array}$

8  $\begin{array}{r|rrrr} 2 & -15 & -11 & 24 & \\ & & 14 & 8 & -24 \\ \hline & 2 & 1 & -3 & \textcircled{0} \end{array}$

$2x^2 + x - 3$   
 $(2x+3)(x-1) = 0$   
 $x = 1$   $x = -\frac{3}{2}$   
 $x = 1$   $x = 8$

Find the zeros of the polynomial.

21.  $P(x) = 6x^4 + 23x^3 + 19x^2 - 8x - 4$

$\frac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}$

-2  $\begin{array}{r|rrrrr} 6 & 23 & 19 & -8 & -4 & \\ & & -12 & -22 & 6 & 4 \\ \hline & 6 & 11 & -3 & -2 & \textcircled{0} \end{array}$

$x = -2$   
 $x = \frac{1}{2}$

$\frac{1}{2}$   $\begin{array}{r|rrrr} 6 & 11 & -3 & -2 & \\ & & 3 & 7 & 2 \\ \hline & 6 & 14 & 4 & \textcircled{0} \end{array}$

$6x^2 + 14x + 4 = 0$   
 $(3x+1)(2x+4) = 0$   
 $x = -\frac{1}{3}$   $x = -2$

Find the zeros of the polynomial.

Hint: Find the rational zeros first.

20.  $P(x) = x^4 + x^3 - 2x^2 + 4x - 24$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

2  $\begin{array}{r|rrrrr} 1 & 1 & -2 & 4 & -24 & \\ & & 2 & 6 & 8 & 24 \\ \hline & 1 & 3 & 4 & 12 & \textcircled{0} \end{array}$

$x^3 + 3x^2 + 4x + 12$

$x = 2$

$x^2(x+3) + 4(x+3)$

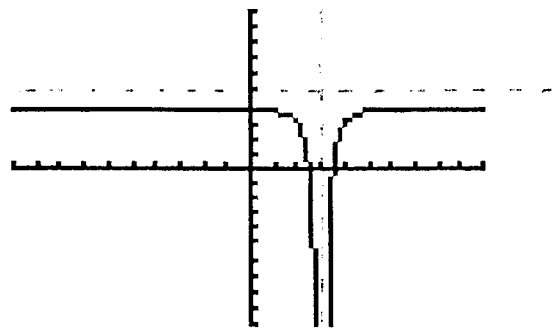
$x = -3$

$(x^2+4)(x+3) = 0$

$x = \pm 2i$

Use stretching/shrinking, reflecting and shifting rules to give an equation of the following graphs.

22.



$f(x) = \frac{-1}{(x-3)^2} + 5$

Give the equations of the vertical, horizontal and/or slant asymptotes of the rational functions.

23.  $f(x) = \frac{2x^2 + 3}{x - 4}$

24.  $f(x) = \frac{2(3x-1)(x+4)}{(x+2)(5x-3)}$

V.A. \_\_\_\_\_ H.A. \_\_\_\_\_ S.A. \_\_\_\_\_

V.A. \_\_\_\_\_ H.A. \_\_\_\_\_ S.A. \_\_\_\_\_

V.A.  $x - 4 = 0$  | H.A. | S.A.

$x = 4$	rule 3 none	$x - 4 \overline{) 2x^2 + 0x + 3}$ $\underline{-2x^2 + 8x}$ $8x + 3$ $\underline{-8x + 32}$ $35$
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V.A.  $(x+2)(5x-3) = 0$  | H.A. Rule 2

$x = -2$	$x = \frac{3}{5}$	$y = \frac{6x^2}{5x^2}$ $y = \frac{6}{5}$
S.A. none		

Find the x-intercepts and y-intercept of the rational function.

25.  $f(x) = \frac{3x}{(x+1)(x-2)}$

26.  $f(x) = \frac{(x-5)(x-2)}{x^2+9}$

x-int \_\_\_\_\_ y-int \_\_\_\_\_

x-int \_\_\_\_\_ y-int \_\_\_\_\_

$y = 0$

$$\frac{0}{3} = \frac{3x}{3}$$

$x = 0$

$(x=0)$

$$f(0) = \frac{3(0)}{(0+1)(0-2)}$$

$y = 0$

$y = 0$

$$0 = (x-5)(x-2)$$

$x = 5$   $x = 2$

$x = 0$

$$f(0) = \frac{(0-5)(0-2)}{0^2+9}$$

$y = \frac{10}{9}$

Sketch the graph and provide information about intercepts and asymptotes.

27.  $f(x) = \frac{x}{x^2 - x - 2}$

See attached sheet

Example Sketch the graph and provide information about intercepts and asymptotes.

$$f(x) = \frac{x}{x^2 - x - 2}$$

1. Find and plot the x-intercepts. (Set numerator = 0 and solve for x)

$$x = 0$$

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2. Find and plot the y-intercepts. (Let x = 0 and solve for y)

$$f(0) = \frac{0}{0^2 - 0 - 2} = 0$$

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3. Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$$\begin{aligned}x^2 - x - 2 &= 0 \\(x + 1)(x - 2) &= 0 \\x &= -1 \text{ and } x = 2\end{aligned}$$

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4. Find and plot the Horizontal Asymptotes. (Top heavy, Bottom heavy or Same)

(Rule 1)  $y = 0$

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5. Find and plot the Slant Asymptotes. (Divide numerator by denominator.)

None

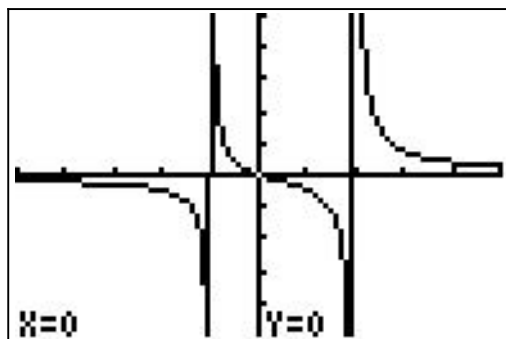
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6. Plot at least one point between and beyond each x-intercept and vertical asymptotes.

choose:

x = -2	x = -.5	x = 1	x = 3
y = -.5	y = .4	y = -.5	y = .75

Note: YOU MAY WANT TO PICK MORE POINTS TO GET A BETTER GRAPH !



ANSWER: