§5.3 Properties of Logarithms

Change of Base Formula :

Let a, b and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then

$$\log_a x = \frac{\log_b x}{\log_b a} \qquad \left(\log_a x = \frac{\log_{10} x}{\log_{10} a} \quad \text{or} \quad \log_a x = \frac{\ln x}{\ln a}\right).$$

Example 1: Changing Bases Using Common Logarithms & Natural Logarithm

a) $\log_4 30$ b) $\log_2 14$ c) $\log_4 30$ d) $\log_2 14$

Properties of Logarithms: (also true for natural logarithms)

1) $\log_a 1 = 0$ because $a^0 = 1$
2) $\log_a a = 1$ because $a^1 = a$
3) $\log_a a^x = x$ because $a^x = a^x$
4) $\log_a x = \log_a y$, then $x = y$

Example 2: Solve for x.

a) $\log_2 x = \log_2 3$ b) $\log_4 4 = x$ c) $\log_2 \frac{1}{8} = x$

Example 3: Rewrite using Properties of Natural Logarithms

a) $\ln \frac{1}{e}$ b) $\ln e^3$ c) $\ln e^0$

Properties of Logarithms:

For any positive real numbers x and y, real number r, and any positive real number a, $(a \neq 1)$:

Product Rule a) $\log_a xy = \log_a x + \log_a y$

Quotient Rule b) $\log_a \frac{x}{y} = \log_a x - \log_a y$

Power Rule c) $\log_a x^r = r \log_a x$

Example 4: Rewrite the logarithm in terms of ln 2 and ln 3.

a) $\ln 6$

b) $\ln \frac{2}{27}$

Example 6: Rewrite using the properties of logarithms.

a) $\log_{10} 5x^3y$

b) $\ln \frac{\sqrt{3x-5}}{7}$

Example 7: Rewrite in condensed form.

a) $\frac{1}{2}\log_{10} x + 3\log_{10}(x+1)$

b) $2\ln(x+2) - \ln x$