Find the Complex Zeros of a Polynomial

Example: Find the complex zeros of:

$$f(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$$

S1) degree = 4

S2 + real 0's

① (1 sign change)

− real 0's

$f(-x) = 3x^4 - 5x^3 + 25x^2 - 45x - 18$

1 1 1 0

(3 or 1) (3 sign changes)

S3) $p = 18 \rightarrow \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$q = 3 \rightarrow \pm 1, \pm 3$

$\frac{p}{q} = \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 3, \pm 6, \pm 9, \pm 18$

S4) choose ~~k=1~~ or ~~k=1~~ (doesn't work remainder ≠ 0)

k = −2 works

$$\begin{array}{c|cccc} -2 & 3 & 5 & 25 & 45 & -18 \\ & & -6 & 2 & -54 & 18 \\ \hline & 3 & -1 & 27 & -9 & 0 \end{array}$$ ✓

S4) continued Depressed 1 equation

then $3x^3 - x^2 + 27x - 9 = 0$

So $x^2(3x-1) + 9(3x-1) = 0$

$(x^2+9)(3x-1) = 0$

$3x - 1 = 0$ | $x^2 + 9 = 0$

$x = \frac{+1}{3}$ | $\sqrt{x^2} = \sqrt{-9}$

$x = \pm 3i$

conjugate pair

zeros:

$x = -2$

$x = \frac{1}{3}$

$x = 3i$

$x = -3i$

factors:

$(x+2)$

$(3x-1)$

$(x-3i)$

$(x+3i)$