

Guidelines for Graphing Rational Functions

example a.) $f(x) = \frac{2(x^2 - 9)}{x^2 - 4}$

1. Find and plot the x-intercepts. (Set numerator = 0 and solve for x).

$$2(x^2 - 9) = 0$$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

2. Find and plot the y-intercepts. (Let $x = 0$ and solve for y)

$$f(0) = \frac{2(0^2 - 9)}{0^2 - 4} = \frac{9}{2}$$

3. Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$$x^2 - 4$$

$$x^2 = 4$$

$$x = \pm 2$$

4. Find and plot the Horizontal Asymptotes. (Top heavy, Bottom heavy or Same)

$$f(x) = \frac{2(x^2 - 9)}{x^2 - 4} \quad \text{Rule 2} \quad \text{Numerator and denominator have the same degree.}$$

$y = 2$ H.A.

5. Find and plot the Slant Asymptotes. (Divide numerator by denominator.)

None ! Only have these if Numerator is exactly 1 degree higher than denominator!

6. Find where the graph will intersect its nonvertical asymptote by solving $f(x) = k$, where k is the y -value of the horizontal asymptote, or $f(x) = mx + b$, where $y = mx + b$ is the equation of the oblique asymptote.

$$\text{Solve } 2 = \frac{2(x^2 - 9)}{x^2 - 4} \quad (\text{No solution!}) \quad \text{No oblique asymptotes.}$$

7. Plot at least one point between and beyond each x-intercept and vertical asymptotes.

Remember Test Points ?

Choose test points carefully!

$x = -4$	$x = -2.5$	$x = 0$	$x = 2.5$	$x = 4$			$x = -1$	$x = 1$	
$y = 1.16$	$y = -2.4$	$y = 4.5$	$y = -2.4$	$y = 1.16$			$y = 5.3$	$y = 5.3$	

Note: YOU STILL MAY HAVE TO PLOT ADDITIONAL POINTS !

Use smooth curves to complete the graph between and beyond the vertical asymptotes.