

§ 1.7 Transformations of Functions (Shifting, Reflecting, and Stretching of Graphs)

Shifting Graphs (Rigid Translations) - given a function $y = f(x)$ and $c > 0$

- (1) the graph of $y = f(x) + c$ is the graph of $y = f(x)$ shifted up c units.
- (2) the graph of $y = f(x) - c$ is the graph of $y = f(x)$ shifted down c units.
- (3) the graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted left c units.
- (4) the graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted right c units.

Example 1: Graph.

a.) $y = |x| + 2$

b.) $y = |x| - 2$

c.) $y = |x + 2|$

d.) $y = |x - 2|$

Reflecting Graphs - given a function $y = f(x)$

- (1) the graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected over the x -axis.
- (2) the graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected over the y -axis.

Example 2: Graph.

a.) $y = -\sqrt{x}$

b.) $y = \sqrt{-x}$

Narrowing and Broadening (Non-Rigid Translations):

The graph of $g(x) = c * f(x)$ has the same general shape as the graph of $f(x)$.

- 1) It is narrowed vertically compared to the graph of $f(x)$ if $c > 1$.
- 2) It is broadened vertically compared to the graph of $f(x)$ if $0 < c < 1$.

Example 3: Graph.

a.) $y = 5|x|$

b.) $y = \frac{1}{2}|x|$

(Note: when an equation contains more than one shifting or reflecting rule, use steps (one rule at a time) and work from the inside of the function to the outside.)