

## § 1.7 Transformations of Functions (Shifting, Reflecting, and Stretching of Graphs)

**Shifting Graphs (Rigid Translations)** - given a function  $y = f(x)$  and  $c > 0$

- (1) the graph of  $y = f(x) + c$  is the graph of  $y = f(x)$  shifted up  $c$  units.
- (2) the graph of  $y = f(x) - c$  is the graph of  $y = f(x)$  shifted down  $c$  units.
- (3) the graph of  $y = f(x + c)$  is the graph of  $y = f(x)$  shifted left  $c$  units.
- (4) the graph of  $y = f(x - c)$  is the graph of  $y = f(x)$  shifted right  $c$  units.

Example 1: Graph.

a.)  $y = |x| + 2$

b.)  $y = |x| - 2$

c.)  $y = |x + 2|$

d.)  $y = |x - 2|$

**Reflecting Graphs** - given a function  $y = f(x)$

- (1) the graph of  $y = -f(x)$  is the graph of  $y = f(x)$  reflected over the  $x$ -axis.
- (2) the graph of  $y = f(-x)$  is the graph of  $y = f(x)$  reflected over the  $y$ -axis.

Example 2: Graph.

a.)  $y = -\sqrt{x}$

b.)  $y = \sqrt{-x}$

## Narrowing and Broadening (Non-Rigid Translations):

The graph of  $g(x) = c * f(x)$  has the same general shape as the graph of  $f(x)$ .

- 1) It is narrowed vertically compared to the graph of  $f(x)$  if  $c > 1$ .
- 2) It is broadened vertically compared to the graph of  $f(x)$  if  $0 < c < 1$ .

Example 3: Graph.    a.)  $y = 5|x|$                       b.)  $y = \frac{1}{2}|x|$

(Note: when an equation contains more than one shifting or reflecting rule, use steps (one rule at a time) and work from the inside of the function to the outside.)