

§3.2 Logarithmic Functions and their Graphs

Logarithm :

For all real numbers y , and all positive numbers a and x , where $a \neq 1$:

$$y = \log_a x \quad \text{if and only if} \quad x = a^y .$$

Note that your calculator has the ability to evaluate two types of logs.

Common Logs	log (base 10)->	log x
Natural Logs	log (base e) ->	ln x

Example 1: Evaluating Logarithms on a Calculator

a) $\log_{10} 2.5$ b) $\log_{10} \left(\frac{1}{3} \right)$ c) $\ln 0.3$ d) $\ln(-1)$

Properties of Logarithms: (also true for natural logarithms)

- 1) $\log_a 1 = 0$ because $a^0 = 1$
- 2) $\log_a a = 1$ because $a^1 = a$
- 3) $\log_a a^x = x$ because $a^x = a^x$
- 4) $\log_a x = \log_a y$, then $x = y$

Graphs of the Form: $f(x) = \log_a x$

- 1) The point $(1, 0)$ is on the graph.
- 2) If $a > 1$, f is an increasing function; If $0 < a < 1$, f is a decreasing function.
- 3) The y -axis is a vertical asymptote.
- 4) The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.

Examples:

a) Graph $y = \log x$

b) Graph $y = \log_2(x - 1) + 3$