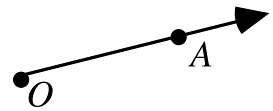
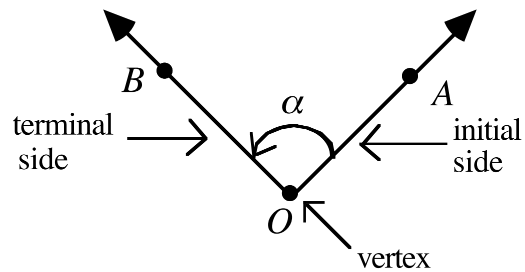
§4.1 Radian and Degree Measure

So what does trigonometry mean ? **Answer: "measurement of triangles!"**

- a ray starts at a point and extends indefinitely

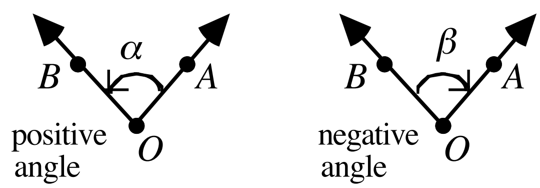


- an angle occurs when a ray is rotated about its endpoint
- the starting position of the ray is the initial side of the angle
- the position of the ray after rotation is the terminal side of the angle
- the meeting point of the two rays is the vertex of the angle



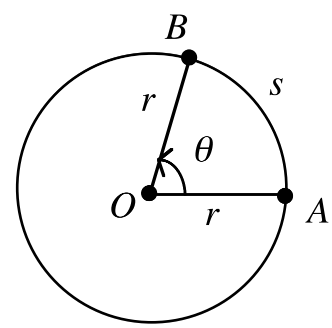
$\angle\alpha = \angle O = \angle AOB$

- a positive angle is formed by a counter-clockwise rotation
- a negative angle is formed by a clockwise rotation
- coterminal angles have the same initial and terminal sides.



Radian Measure

- consider a circle of radius r with two radii OA and OB
- the angle θ formed by these two radii is a central angle
- the arc AB is the part of the circle between A and B and its length is s
- the arc AB subtends the angle θ

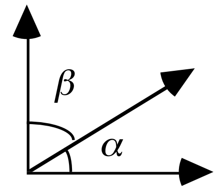


- the measure of the central angle subtended by an arc of length r on a circle with radius r is one radian

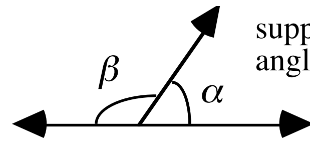
- the radian measure of the central angle subtended by an arc of length s on a circle of radius r is $\theta = \frac{s}{r}$ or $s = r\theta$

- given a circle of radius r, the radian measure of the central angle subtended by the circumference of the circle is $\theta = \frac{2\pi r}{r} = 2\pi$ while in degrees $\theta = 360°$
- thus, $360° = 2\pi$ radians and $180° = \pi$ radians

- two **nonnegative** angles α and β are <u>complementary angles</u> if $\alpha + \beta = 90°$
- in this case, α is the <u>complement</u> of β and vice versa
- two **nonnegative** angles α and β are <u>supplementary angles</u> if $\alpha + \beta = 180°$
- in this case, α is the <u>supplement</u> of β and vice versa



complementary angles



supplementary angles

- Discuss common angles, revolutions, and quadrants in the coordinate system.

Example 1 Find the coterminal angles for the following angles.

a) $\dfrac{13\pi}{6}$ b) $\dfrac{-2\pi}{3}$

Example 2 Find the complement and supplement angles for the following angles.

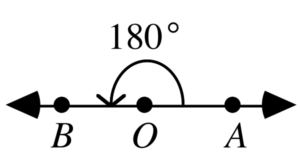
a) $\dfrac{2\pi}{5}$ b) $\dfrac{4\pi}{5}$

Degree Measure

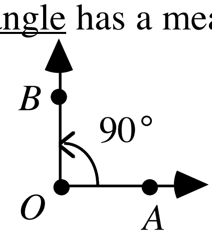
- an angle formed by rotating a ray $\frac{1}{360}$ of a complete revolution has a measure of 1 <u>degree</u> $(1°)$

- angles are often classified by their measures

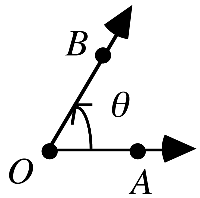
(1) a <u>straight angle</u> has a measure of $180°$



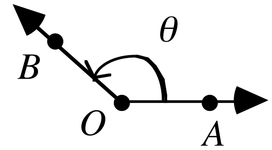
(2) a <u>right angle</u> has a measure of $90°$



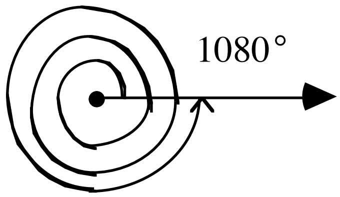
(3) an <u>acute angle</u> has a measure
$0° < \theta < 90°$

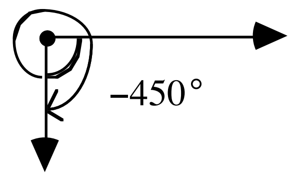


(4) an <u>obtuse angle</u> has a measure
$90° < \theta < 180°$



- angles larger than $360°$ or smaller than $-360°$ can be measured by considering more than one rotation





Radian-Degree Conversion Factors

- to change radians to degrees, multiply the number of radians by $\dfrac{180°}{\pi}$
- to change degrees to radians, multiply the number of degrees by $\dfrac{\pi}{180°}$

Example Convert from Degrees to Radians.

a) $135°$ b) $540°$ c) $-270°$

Example Convert from Radians to Degrees

a) $\dfrac{-\pi}{2}$ b) $\dfrac{9\pi}{2}$ c) 2 radians

<u>**DMS System (Degree, Minute, Second)**</u>

1 minute $(1') = \left(\frac{1}{60}\right)°$ $\Rightarrow 60' = 1°$

1 second $(1'') = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)°$ $\Rightarrow 60'' = 1'$ and $3600'' = 1°$

Example Convert $20° 4' 45''$ to decimal degree measure to the nearest thousandth.

Example Convert $342.17°$ to DMS.

Example Find the complement and supplement of $19° 42' 05''$.

Note: You **MUST** memorize all degree to radian conversions of the selected angles listed below and know their positions on a circle measured from the positive x-axis.

Degrees	Radians
0	0
30	$\pi/6$
45	$\pi/4$
60	$\pi/3$
90	$\pi/2$
120	$2\pi/3$
135	$3\pi/4$
150	$5\pi/6$
180	π
210	$7\pi/6$
225	$5\pi/4$
240	$4\pi/3$
270	$3\pi/2$
300	$5\pi/3$
315	$7\pi/4$
330	$11\pi/6$
360	2π