

§4.2 Trigonometric Functions: The Unit Circle

Discuss the Unit Circle.

The Trigonometric Functions

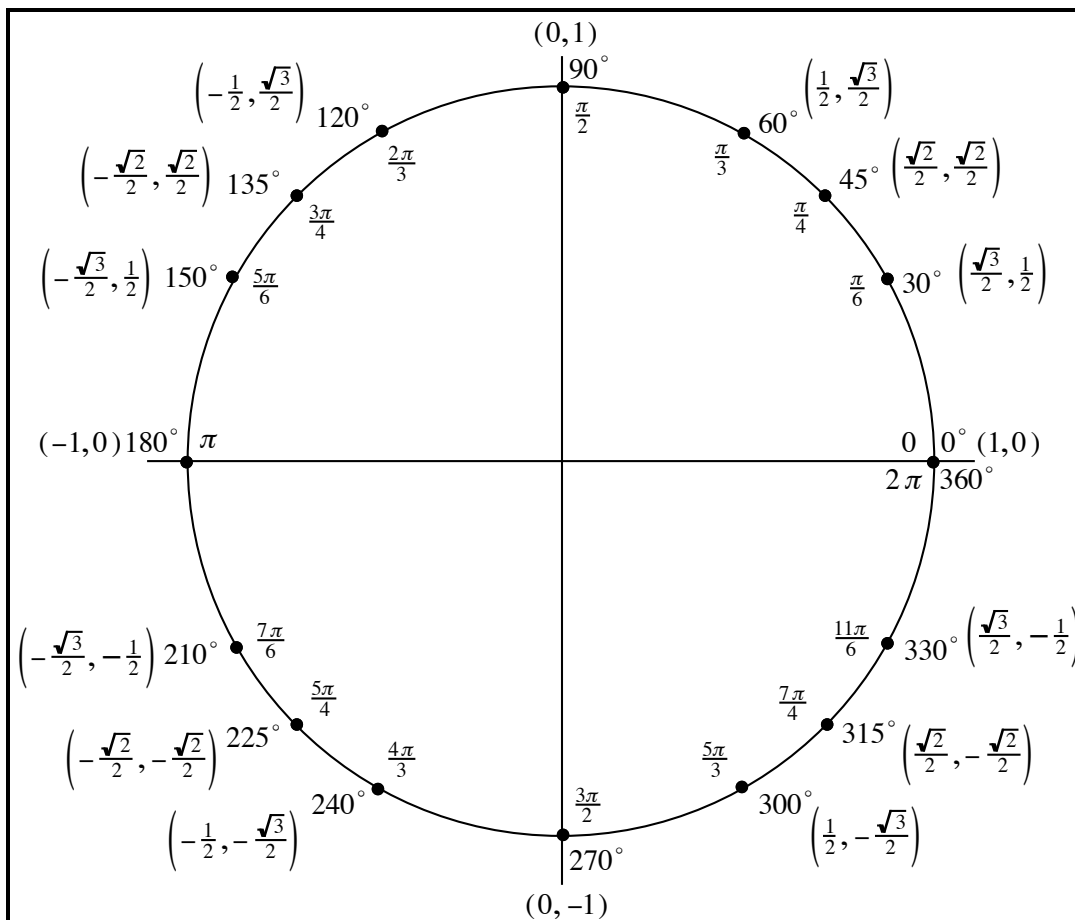
Let t be a real number and let (x, y) be the point on the unit circle corresponding to t .

$$\sin t = y \qquad \csc t = \frac{1}{y} \quad (y \neq 0)$$

$$\cos t = x \qquad \sec t = \frac{1}{x} \quad (x \neq 0)$$

$$\tan t = \frac{y}{x} \quad (x \neq 0) \qquad \cot t = \frac{x}{y} \quad (y \neq 0)$$

The Unit Circle



$$(x, y) = (\cos \theta, \sin \theta)$$

Example 1 Evaluate the six trig functions at each real number.

a) $t = \frac{\pi}{6}$

b) $t = \frac{5\pi}{4}$

c) $t = \pi$

Domain(Range) and Period of Sine and Cosine

Domain of the Sine and Cosine is all real numbers: $-\infty < t < \infty$

Range of Sine and Cosine: $-1 \leq y \leq 1$ and $-1 \leq x \leq 1$
 $-1 \leq \sin t \leq 1$ and $-1 \leq \cos t \leq 1$

Definition of a periodic function

A function f is **periodic** if there exists a positive real number c such that

$$f(t + c) = f(t)$$

for all t in the domain of f . The smallest number c for which f is periodic is called the **period** of f .

Examples: a) Find $\sin \frac{13\pi}{6}$ b) Find $\cos -\frac{7\pi}{2}$

Even and Odd Trigonometric Functions

The cosine and secant functions are **even**.

$$\cos(-t) = \cos t \qquad \sec(-t) = \sec t$$

The sine, cosecant, tangent, and cotangent functions are **odd**.

$$\sin(-t) = -\sin(t) \qquad \csc(-t) = -\csc(t)$$

$$\tan(-t) = -\tan(t) \qquad \cot(-t) = -\cot(t)$$

Discuss using a calculator.