

## §4.4 Trigonometric Functions of Any Angle

### Definitions of Trigonometric Functions of Any Angle

Let  $\theta$  be an angle in standard position with  $(x, y)$  a point on the terminal side of  $\theta$  and  $r = \sqrt{x^2 + y^2} \neq 0$ .

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

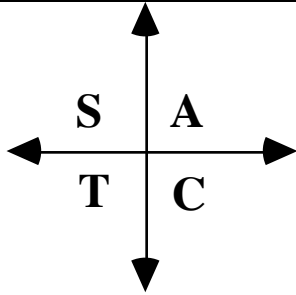
$$\tan \theta = \frac{y}{x}, \quad (x \neq 0)$$

$$\csc \theta = \frac{r}{y}, \quad (y \neq 0)$$

$$\sec \theta = \frac{r}{x}, \quad (x \neq 0)$$

$$\cot \theta = \frac{x}{y}, \quad (y \neq 0)$$

#### Signs of Trigonometric Functions



"All Students Take Calculus"

Quadrant I  $\Rightarrow$   $\sin \theta$ ,  $\cos \theta$ ,  $\tan \theta$  are positive

Quadrant II  $\Rightarrow$   $\sin \theta$  is positive;  $\cos \theta$ ,  $\tan \theta$  are negative

Quadrant III  $\Rightarrow$   $\tan \theta$  is positive;  $\sin \theta$ ,  $\cos \theta$  are negative

Quadrant IV  $\Rightarrow$   $\cos \theta$  is positive;  $\sin \theta$ ,  $\tan \theta$  are negative

Example 1 Let  $(-3, 4)$  be a point on the terminal side of  $\theta$ . Find the sine, cosine, and tangent of  $\theta$ .

Example 2: Given  $\tan \theta = \frac{-5}{4}$  and  $\cos \theta > 0$ , find  $\sin \theta$  and  $\sec \theta$ .

Example 3: Evaluate the sine function at the four quadrant angles  $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ .

## Reference Angles

Let  $\theta$  be an angle in standard position. Its **reference angle** is the acute angle  $\theta'$  formed by the terminal side of  $\theta$  and the horizontal axis.

Example 4: Find the reference angle  $\theta'$ .

a)  $\theta = 300^\circ$

b)  $\theta = 2.3$

c)  $\theta = -135^\circ$

## Evaluating Trigonometric Functions of Any Angle

- 1) determine the function value for the associated reference angle  $\theta'$ .
- 2) depending on the quadrant in which  $\theta$  lies, affix the appropriate sign to the function value.

Example 5: Evaluate each trig function.

a)  $\cos \frac{4\pi}{3}$

b)  $\tan(210^\circ)$

c)  $\csc = \frac{11\pi}{4}$

Example 6: Let  $\theta$  be an angle in Quadrant II such that  $\sin\theta = \frac{1}{3}$ ,  
by using trigonometric identities find:

a)  $\cos\theta$

b)  $\tan\theta$

Example 7: Use a calculator to find:

a)  $\cot 410^\circ$  and  $\sin(-7)$

b) Solve  $\tan\theta = 4.812$ ,  
 $0 \leq \theta < 2\pi$