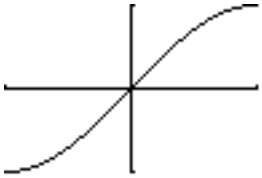
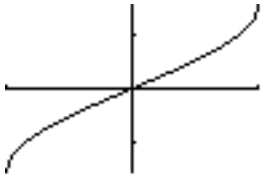
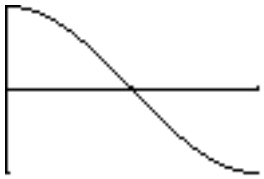
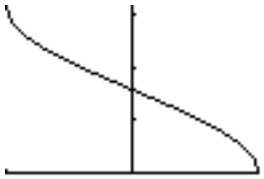
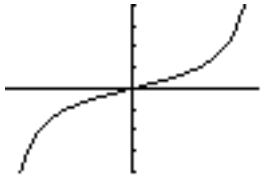
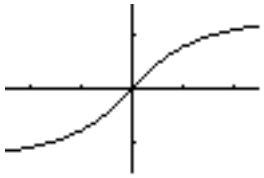
§4.7 Inverse Trigonometric Functions

- the function $y = \sin x$ is not one-to-one since its graph fails the horizontal line test

- by restricting the domain of $y = \sin x$ to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, it is one-to-one and has an inverse function $y = \arcsin x$

Function	Inverse Function	Restrictions
		$y = \arcsin x$ $x = \sin y$ $-1 \le x \le 1$ (domain) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (range)
		$y = \arccos x$ $x = \cos y$ $-1 \le x \le 1$ (domain) $0 \le y \le \pi$ (range)
		$y = \arctan x$ $x = \tan y$ $-\infty < x < \infty$ (domain) $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (range)

Note: Another notation for $y = \arcsin x$ is $\sin^{-1} x \neq \frac{1}{\sin x} = (\sin x)^{-1}$

Note: in $y = \sin^{-1} x$, y is the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is x.

Example 1 Evaluate.

a.) $\arcsin\left(\frac{-1}{2}\right)$ b.) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ c.) $\sin^{-1}(2)$

Example 2 Evaluate.

a.) $\arccos\left(\frac{\sqrt{2}}{2}\right)$ b.) $\cos^{-1}(-1)$ c.) $\arctan(0)$

Example 3 Evaluate. (use calculator)

a.) $\tan^{-1}(-8.45)$ b.) $\sin^{-1}(0.2447)$ c.) $\arccos(2)$

Compositions of Functions

If possible find the exact value.

Example 4 Find the radian value to four decimal places.

a.) $\tan\left[\arctan(-5)\right]$ b.) $\arcsin\left(\sin\frac{5\pi}{3}\right)$ c.) $\cos\left(\cos^{-1}\pi\right)$

Example 5 Evaluate.

a.) $\tan\left(\arccos\frac{2}{3}\right)$ b.) $\cos\left[\arcsin\left(\frac{-3}{5}\right)\right]$