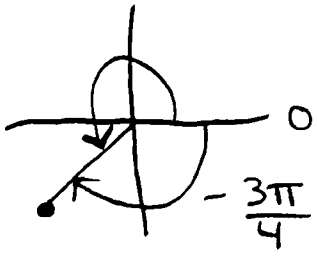
Math 1113 Sample Test 2 Name Answer Key

Precalculus Sections 4.1 - 4.5 Date _____

Directions. Show all work. Circle final answers.

Find a coterminal angle for the following angle.

1. $\frac{-3\pi}{4}$



$2\pi - \frac{3\pi}{4}$

$= \frac{8\pi}{4} - \frac{3\pi}{4}$

$= \frac{5\pi}{4}$

many answers

Find the complement angle of the following.

2. $\frac{2\pi}{5}$ must sum to 90°

$\frac{\pi}{2} - \frac{2\pi}{5} = \frac{5\pi}{10} - \frac{4\pi}{10} = \frac{\pi}{10}$

or $90^\circ - 72^\circ = 18^\circ$

$\frac{2\pi}{5} \cdot \frac{180}{\pi} = 72^\circ$

Convert from Radians to Degrees.

3. $\frac{9\pi}{2} \cdot \frac{90}{180} = 810^\circ$

A circle has radius of 4 inches. Find the arc length for the following angle. ($s = r\theta$)

4. 120° must convert to radians

$120^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{3}$ θ must be in radians

$s = r\theta$

$= 4\frac{2\pi}{3} = \frac{8\pi}{3}$

5. Evaluate the six trig functions for the following angle. $\frac{-\pi}{3}$

$\sin\left(\frac{-\pi}{3}\right) = \sin(-60) = \frac{-\sqrt{3}}{2}$

$\cos\left(\frac{-\pi}{3}\right) = \cos(-60) = \frac{+1}{2}$

$\tan\left(\frac{-\pi}{3}\right) = \tan(-60) = -\sqrt{3}$

$\csc\left(\frac{-\pi}{3}\right) = \csc(-60) = \frac{-2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

$\sec\left(\frac{-\pi}{3}\right) = \sec(-60) = 2$ $\cot\left(\frac{-\pi}{3}\right) = \frac{-1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

6. List which trig functions are even and which are odd.

Even: cos, sec

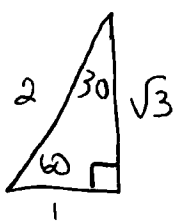
Odd: sin, tan, csc

7. Find the following values of $\sin 45^\circ$, $\cos 60^\circ$, and $\tan 30^\circ$.

$\sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$



8. Let θ be an acute angle such that $\tan\theta = 3$. Find the value of $\sec\theta$.

$\tan^2\theta + 1 = \sec^2\theta$

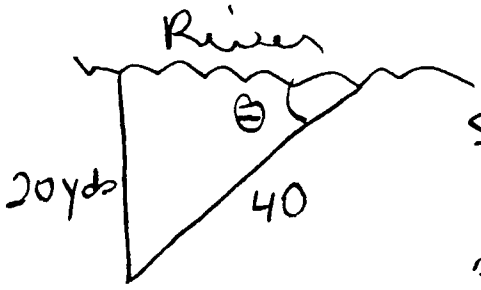
$3^2 + 1 = \sec^2\theta$

$9 + 1 = \sec^2\theta$

$10 = \sec^2\theta$

$\sec\theta = +\sqrt{10}$ (acute)

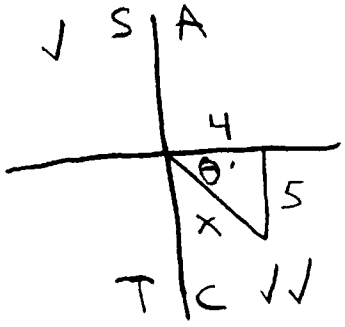
9. You are 20 yards from a river. Rather than walking directly to the river, you walk 40 yards along a straight path to the rivers edge. Find the acute angle θ between this path and the river's edge.



$\sin\theta = \frac{opp}{hyp} = \frac{20}{40} = \frac{1}{2}$

$20\ \theta = \boxed{30^\circ}$

10. Given $\tan\theta = \frac{-5}{4}$ and $\cos\theta > 0$, find $\sin\theta$ and $\sec\theta$. (Hint: Draw a picture.)



$x^2 = 4^2 + 5^2$

$x^2 = 16 + 25$

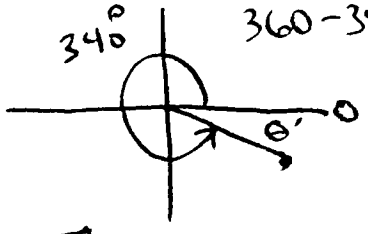
$x = \sqrt{41}$

$\sin\theta = \frac{-5}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}}$

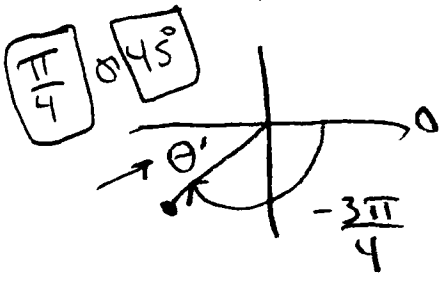
$= \frac{-5\sqrt{41}}{41}$

$\sec\theta = \frac{\sqrt{41}}{4}$

11. Find the reference angle for $\theta = 340^\circ$ and $\theta = -\frac{3\pi}{4}$



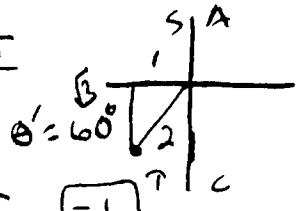
$360 - 340 = \boxed{20^\circ}$



12. Evaluate each trig function. $\cos\frac{4\pi}{3}$ and $\tan(-210^\circ)$.

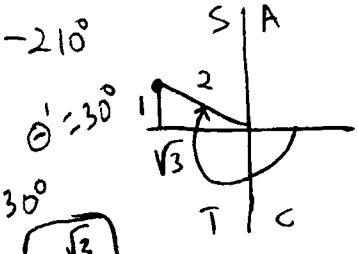
(Hint: Remember All Students Take Calculus.)

$\cos\frac{4\pi}{3}$



$\cos\frac{4\pi}{3} = \boxed{\frac{-1}{2}}$

$\tan -210^\circ$

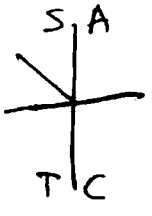


$\tan 30^\circ$

$= \boxed{\frac{-\sqrt{3}}{3}}$

TOA

13. Let θ be an angle in Quadrant II such that $\sin\theta = \frac{1}{3}$, by using trigonometric identities find: $\cos\theta$.



$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$

$\frac{1}{9} + \cos^2\theta = 1$

$\cos^2\theta = 1 - \frac{1}{9}$

$\cos^2\theta = \frac{9}{9} - \frac{1}{9}$

$\cos^2\theta = \frac{8}{9}$

$\cos\theta = \boxed{\frac{-2\sqrt{2}}{3}}$

In order to receive full credit for a graph, you must do all of the following.
1.) Label your axes.
2.) Show at least one period.
3.) Label five ordered pairs or asymptotes (as appropriate).

14. Graph the following function: $y = -3\sin x$

See attached sheet

15. Graph the following function: $y = 2 + 3\cos(2x)$

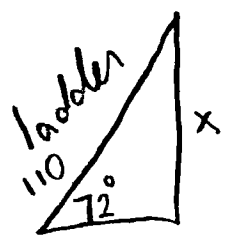
See attached sheet

16. ~~Graph the following function: $y = \tan\frac{x}{2}$~~

17. ~~Graph the following function: $y = 2\cot\frac{x}{3}$~~

18. ~~Graph the following function: $y = 2\csc\left[x + \frac{\pi}{4}\right]$~~

19. A safety regulation states that the maximum angle of elevation for a rescue ladder is 72°. A fire department's longest ladder is 110 feet. What is the maximum safe rescue height?



$\sin 72^\circ = \frac{x}{110}$

$x = 110\sin 72^\circ = 104.6$ feet

Answers Sample Test 2

1. $\frac{5\pi}{4}$ (Note: there many answers possible.)	2. $\frac{\pi}{10}$
3. 810°	4. $\frac{8\pi}{3}$
5. $\sin\left(\frac{-\pi}{3}\right)=\frac{-\sqrt{3}}{2}$ $\csc\left(\frac{-\pi}{3}\right)=\frac{-2\sqrt{3}}{3}$ $\cos\left(\frac{-\pi}{3}\right)=\frac{1}{2}$ $\sec\left(\frac{-\pi}{3}\right)=2$ $\tan\left(\frac{-\pi}{3}\right)=-\sqrt{3}$ $\cot\left(\frac{-\pi}{3}\right)=\frac{-\sqrt{3}}{3}$	6. Even -> cos and sec Odd -> sin, tan, csc, cot
7. $\sin 45^\circ=\frac{\sqrt{2}}{2}$, $\cos 60^\circ=\frac{1}{2}$, $\tan 30^\circ=\frac{\sqrt{3}}{3}$	8. $\sec\theta=\sqrt{10}$
9. $\theta=30^\circ$	10. $\sin\theta=\frac{y}{r}=\frac{-5}{\sqrt{41}}$ $\sec\theta=\frac{r}{x}=\frac{\sqrt{41}}{4}$
11. $\theta=340^\circ$ -> $\theta'=20^\circ$ $\theta=-\frac{3\pi}{4}$ -> $\theta'=45^\circ$	12. $\cos\frac{4\pi}{3}=\frac{-1}{2}$ (Quadrant III) $\tan(-210^\circ)=\frac{-\sqrt{3}}{3}$ (Quadrant II)
13. $\cos\theta=\frac{-2\sqrt{2}}{3}$	

14. $y = -3\sin x$

(Remember APTEV)

amplitude = $|a| = |-3| = 3$

period (of sine and cosine) = $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

tick marks = $\frac{\text{period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

tick mark calculations:

(1) 0

(2) $0 + \frac{\pi}{2} = \frac{\pi}{2}$

(3) $\frac{\pi}{2} + \frac{\pi}{2} = \pi$

(4) $\pi + \frac{\pi}{2} = \frac{3\pi}{2}$

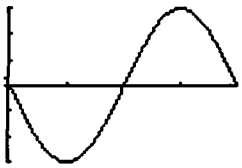
(5) $\frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

endpoints Solve:

$bx - c = 0$ $bx - c = 2\pi$

$x = 0$ $x = 2\pi$

(starts) (ends)



vertical shift = d = none

15. $y = 3\cos(2x) + 2$ (Remember APTEV)

amplitude = $|a| = |3| = 3$

period (of sine and cosine) = $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

tick marks = $\frac{\text{period}}{4} = \frac{\pi}{4}$

tick mark calculations:

(1) 0

(2) $0 + \frac{\pi}{4} = \frac{\pi}{4}$

(3) $\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

(4) $\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

(5) $\frac{3\pi}{4} + \frac{\pi}{4} = \frac{4\pi}{4} = \pi$

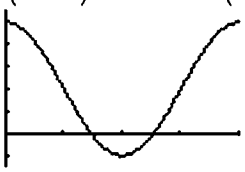
endpoints Solve:

$bx - c = 0$ $bx - c = 2\pi$

$2x - 0 = 0$ $2x - 0 = 2\pi$

$x = 0$ $x = \pi$

(starts) (ends)



vertical shift = d = 2

16. $y = \tan\left(\frac{x}{2}\right)$ (Remember APTEV)

Formulas for General Form $y = a\tan(bx - c) + d$

amplitude = none

period (of tan and cot) = $\frac{\pi}{b} = \frac{\pi}{1/2} = 2\pi$

tick marks = $\frac{\text{period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

tick mark calculations:

(1) $-\pi$

(2) $-\pi + \frac{\pi}{2} = \frac{-\pi}{2}$

(3) $\frac{-\pi}{2} + \frac{\pi}{2} = 0$

(4) $0 + \frac{\pi}{2} = \frac{\pi}{2}$

(5) $\frac{\pi}{2} + \frac{\pi}{2} = \pi$

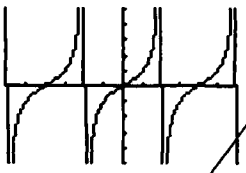
endpoints Solve:

$bx - c = \frac{-\pi}{2}$ $bx - c = \frac{\pi}{2}$

$\frac{x}{2} = \frac{-\pi}{2}$ $\frac{x}{2} = \frac{\pi}{2}$

$x = -\pi$ $x = \pi$

(starts) (ends)



vertical shift = none

17. $y = 2\cot\left(\frac{x}{3}\right)$ (Remember APTEV)

Formulas for General Form $y = a\ \cot(bx - c) + d$

amplitude = none

period (of tan and cot) = $\frac{\pi}{b} = \frac{\pi}{1/3} = 3\pi$

tick marks = $\frac{\text{period}}{4} = \frac{3\pi}{4}$

tick mark calculations:

(1) 0

(2) $0 + \frac{3\pi}{4} = \frac{3\pi}{4}$

(3) $\frac{3\pi}{4} + \frac{3\pi}{4} = \frac{3\pi}{2}$

(4) $\frac{3\pi}{2} + \frac{3\pi}{4} = \frac{9\pi}{4}$

(5) $\frac{9\pi}{4} + \frac{3\pi}{4} = 3\pi$

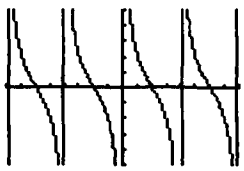
endpoints Solve:

$bx - c = 0$ $bx - c = \pi$

$\frac{x}{3} = 0$ $\frac{x}{3} = \pi$

$x = 0$ $x = 3\pi$

(starts) (ends)



vertical shift = none

18. $y = 2\csc\left(x + \frac{\pi}{4}\right)$ (Remember APTEV)

amplitude = $|a| = |2| = 2$

period (of sine and cosine) = $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

tick marks = $\frac{\text{period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

tick mark calculations:

(1) $\frac{-\pi}{4}$

(2) $\frac{-\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$

(3) $\frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$

(4) $\frac{3\pi}{4} + \frac{\pi}{2} = \frac{5\pi}{4}$

(5) $\frac{5\pi}{4} + \frac{\pi}{2} = \frac{7\pi}{4}$

endpoints Solve:

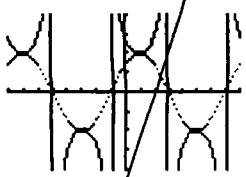
$bx - c = 0$ $bx - c = 2\pi$

$x + \frac{\pi}{4} = 0$ $x + \frac{\pi}{4} = 2\pi$

$x = \frac{-\pi}{4}$ $x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

(starts) (ends)

Remember to graph: $y = 2\sin\left(x + \frac{\pi}{4}\right)$



vertical shift = d = none

19. 104.6 feet