§5.5 Multiple-Angle and Product-to-Sum Formulas

REMEMBER YOU KNOW ALGEBRA !

Double-Angle Identities

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= 1 - 2\sin^2\alpha$

$= 2\cos^2\alpha - 1$

$\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

Half-Angle Identities

$\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

$\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$

$\tan\dfrac{\alpha}{2} = \dfrac{\sin\alpha}{1+\cos\alpha} = \dfrac{1-\cos\alpha}{\sin\alpha}$

Example 1 Evaluate.

Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$ from $\cos\theta = \dfrac{5}{13}$, $\dfrac{3\pi}{2} < \theta < 2\pi$

Example 2 Find the exact value.

$\sin 105^\circ$

Product to Sum Identities

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$

$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$

$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$

$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$

Example 3 Rewrite as a sum or difference. $\cos 5x \sin 4x$

Sum-to-Product Identities

$\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

$\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

$\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

$\cos x - \cos y = -2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

Example 4 Find the exact value of $\cos 195^\circ + \cos 105^\circ$

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