Math 1113 Sample Quiz/Test 7 Name ____________________________

Precalculus Sections 5.1-5.5 Date ____________________________

Directions. Show all work. Circle final answers.

1. Use the values $\tan x = \frac{5}{12}$ and $\sec x = \frac{-13}{12}$ to find the values of all six trig functions.

2. Use the fundamental identities to simplify the expression. $\frac{\cos^2 y}{1 - \sin y}$

3. Factor the expression and use the fundamental identities to simplify.: $\sin^4 x - \cos^4 x$

4. Use the fundamental identities to simplify: $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}$

5. Simplify: $(1 + \sin\alpha)(1 - \sin\alpha)$

6. Verify the identity: $\frac{1}{\sec x \tan x} = \csc x - \sin x$

7. Verify the identity: $\frac{\csc(-x)}{\sec(-x)} = -\cot x$

8. Verify the identity:
$\frac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$

9. Verify that the x-values are solutions of the equation. $2\cos x - 1 = 0$

a) $\frac{\pi}{3}$ b) $\frac{5\pi}{3}$

10. Solve the equation. $3\sec^2 x - 4 = 0$
Find **ALL SOLUTIONS** !

11. Find ALL solutions of the equation in the interval $[0,2\pi)$.

$3\tan^3 x = \tan x$

12. Solve the equation. $\cos 2x = \frac{1}{2}$

Find **ALL SOLUTIONS** !

13. Find the exact values of the sine, cosine, and tangent of the angle.

$105° = 60° + 45°$

14. Find the exact values of the sine, cosine, and tangent of the angle.

$\frac{13\pi}{12}$

15. Write the expression as the sine, cosine, or tangent of an angle.

$$\frac{\tan 325^\circ - \tan 86^\circ}{1 + \tan 325^\circ \tan 86^\circ}$$

16. Find the exact values of the trig function given that $\sin u = \frac{-7}{25}$ and $\cos uv = \frac{-4}{5}$. Both u and v are in Quadrant III.

$\cos(u + v)$

17. Use the DOUBLE-ANGLE formula to rewrite the expression

$4 - 8\sin^2 x$

18. Find the exact values of $\sin\left(\frac{u}{2}\right), \cos\left(\frac{u}{2}\right)$, $\tan\left(\frac{u}{2}\right)$ using the HALF-ANGLE formulas.

$\sin u = \frac{5}{13}$ $\quad\left(\frac{\pi}{2} < u < \pi\right)$

19. Use the product-to-sum formulas to write the product as a sum or difference.

$6\sin\frac{\pi}{4}\cos\frac{\pi}{4}$

20. Use the sum-to- product formulas to write the sum or difference as a product.

$\cos 6x + \cos 2x$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient or Ratio Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Cofunction Identities

$$\sin(90^\circ - \theta) = \cos\theta \qquad \cos(90^\circ - \theta) = \sin\theta \qquad \tan(90^\circ - \theta) = \cot\theta$$

$$\csc(90^\circ - \theta) = \sec\theta \qquad \sec(90^\circ - \theta) = \csc\theta \qquad \cot(90^\circ - \theta) = \tan\theta$$

Even and Odd Trigonometric Functions

The cosine and secant functions are **<u>even</u>**.

$\cos(-t) = \cos t$ $\qquad$ $\sec(-t) = \sec t$

The sine, cosecant, tangent, and cotangent functions are **<u>odd</u>**.

$\sin(-t) = -\sin(t)$ $\qquad$ $\csc(-t) = -\csc(t)$

$\tan(-t) = -\tan(t)$ $\qquad$ $\cot(-t) = -\cot(t)$

Sum or Difference of Two Angles Identities
$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
$\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Double-Angle Identities	
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$	$\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$
$= 1 - 2\sin^2\alpha$	
$= 2\cos^2\alpha - 1$	

Half-Angle Identities	
$\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$	$\tan\dfrac{\alpha}{2} = \dfrac{\sin\alpha}{1+\cos\alpha} = \dfrac{1-\cos\alpha}{\sin\alpha}$
$\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$	

Product to Sum Identities	
$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$	$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$
$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$	$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$

Sum-to-Product Identities	
$\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$	$\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$
$\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$	$\cos x - \cos y = -2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

Answers Sample Test 3

1.

$\tan x = \frac{5}{12}$, $\sec x = -\frac{13}{12} \Rightarrow x$ is in Quadrant III.

$$\cos x = \frac{1}{\sec x} = -\frac{12}{13}$$

$$\sin x = -\sqrt{1 - \cos^2 x} = -\sqrt{1 - \frac{144}{169}} = -\frac{5}{13}$$

$$\cot x = \frac{1}{\tan x} = \frac{12}{5}$$

$$\csc x = \frac{1}{\sin x} = -\frac{13}{5}$$

2.

$$\frac{\cos^2 y}{1 - \sin y} = \frac{1 - \sin^2 y}{1 - \sin y} = \frac{(1 + \sin y)(1 - \sin y)}{1 - \sin y} = 1 + \sin y$$

3.

$$\begin{aligned}\sin^4 x - \cos^4 x &= (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)\\ &= (1)(\sin^2 x - \cos^2 x)\\ &= \sin^2 x - \cos^2 x\end{aligned}$$

4.

$$\begin{aligned}\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} &= \frac{1 - \cos x + 1 + \cos x}{(1 + \cos x)(1 - \cos x)}\\ &= \frac{2}{1 - \cos^2 x}\\ &= \frac{2}{\sin^2 x}\\ &= 2\csc^2 x\end{aligned}$$

5. $(1 + \sin\alpha)(1 - \sin\alpha) = 1 - \sin^2\alpha = \cos^2\alpha$

6.

$$\begin{aligned}\frac{1}{\sec x \tan x} = \cos x \cot x = \cos x \cdot \frac{\cos x}{\sin x} &= \frac{\cos^2 x}{\sin x}\\ &= \frac{1 - \sin^2 x}{\sin x}\\ &= \frac{1}{\sin x} - \sin x\\ &= \csc x - \sin x\end{aligned}$$

7.

$$\begin{aligned}\frac{\csc(-x)}{\sec(-x)} &= \frac{1/\sin(-x)}{1/\cos(-x)}\\ &= \frac{\cos(-x)}{\sin(-x)}\\ &= \frac{\cos x}{-\sin x}\\ &= -\cot x\end{aligned}$$

8.

$$\begin{aligned}\frac{\tan x + \cot y}{\tan x \cot y} &= \frac{\frac{1}{\cot x} + \frac{1}{\tan y}}{\frac{1}{\cot x} \cdot \frac{1}{\tan y}} \cdot \frac{\cot x \tan y}{\cot x \tan y}\\ &= \tan y + \cot x\end{aligned}$$

9. $2\cos x - 1 = 0$ (a) $2\cos\frac{\pi}{3} - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$ (b) $2\cos\frac{5\pi}{3} - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$	**10.** $3\sec^2 x - 4 = 0$ $\sec^2 x = \frac{4}{3}$ $\sec x = \pm\frac{2}{\sqrt{3}}$ $x = \frac{\pi}{6} + n\pi$ or $x = \frac{5\pi}{6} + n\pi$
11. $3\tan^3 x - \tan x = 0$ $\tan x(3\tan^2 x - 1) = 0$ $\tan x = 0$ or $3\tan^2 x - 1 = 0$ $x = 0, \pi$ $\quad$ $\tan x = \pm\frac{\sqrt{3}}{3}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	**12.** $\cos 2x = \frac{1}{2}$ $2x = \frac{\pi}{3} + 2n\pi$ or $2x = \frac{5\pi}{3} + 2n\pi$ $x = \frac{\pi}{6} + n\pi$ $\quad$ $x = \frac{5\pi}{6} + n\pi$
13. $\sin 105° = \sin(60° + 45°)$ $= \sin 60° \cos 45° + \cos 60° \sin 45°$ $= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$ $= \frac{\sqrt{2}}{4}(\sqrt{3} + 1)$ $\cos 105° = \cos(60° + 45°)$ $= \cos 60° \cos 45° - \sin 60° \sin 45°$ $= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$ $= \frac{\sqrt{2}}{4}(1 - \sqrt{3})$ $\tan 105° = \tan(60° + 45°)$ $= \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$ $= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}\cdot\frac{1 + \sqrt{3}}{1 + \sqrt{3}}$ $= \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$	**14.** $\frac{13\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{3}$ $\sin\frac{13\pi}{12} = \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$ $= \sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \cos\frac{3\pi}{4}\sin\frac{\pi}{3}$ $= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{2}}{4}(1 - \sqrt{3})$ $\cos\frac{13\pi}{12} = \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$ $= \cos\frac{3\pi}{4}\cos\frac{\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{\pi}{3}$ $= -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = -\frac{\sqrt{2}}{4}(1 + \sqrt{3})$ $\tan\frac{13\pi}{12} = 2 - \sqrt{3}$

15.

$$\frac{\tan 325° - \tan 86°}{1 + \tan 325° \tan 86°} = \tan(325° - 86°) = \tan 239°$$

16.

$\sin u = -\frac{7}{25}$, u **in Quadrant III** $\Longrightarrow \cos u = -\frac{24}{25}$, $\tan u = \frac{7}{24}$

$\cos v = -\frac{4}{5}$, v **in Quadrant III** $\Longrightarrow \sin v = -\frac{3}{5}$, $\tan v = \frac{3}{4}$

$$\begin{aligned}\cos(u + v) &= \cos u \cos v - \sin u \sin v \\ &= \left(-\tfrac{24}{25}\right)\left(-\tfrac{4}{5}\right) - \left(-\tfrac{7}{25}\right)\left(-\tfrac{3}{5}\right) \\ &= \tfrac{3}{5}\end{aligned}$$

17.

$$\begin{aligned}4 - 8\sin^2 x &= 4(1 - 2\sin^2 x) \\ &= 4\cos 2x\end{aligned}$$

18.

$$\sin u = \frac{5}{13},\ \frac{\pi}{2} < u < \pi \Longrightarrow \cos u = -\frac{12}{13}$$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + \frac{12}{13}}{2}} = \frac{5\sqrt{26}}{26}$$

$$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 - \frac{12}{13}}{2}} = \frac{\sqrt{26}}{26}$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1 + \cos u} = \frac{\frac{5}{13}}{1 - \frac{12}{13}} = 5$$

19.

$$6\sin\frac{\pi}{4}\cos\frac{\pi}{4} = 6 \cdot \frac{1}{2}\left[\sin\left(\frac{\pi}{4} + \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4} - \frac{\pi}{4}\right)\right] = 3\left(\sin\frac{\pi}{2} + \sin 0\right)$$

20.

$$\cos 6x + \cos 2x = 2\cos\left(\frac{6x + 2x}{2}\right)\cos\left(\frac{6x - 2x}{2}\right) = 2\cos 4x \cos 2x$$