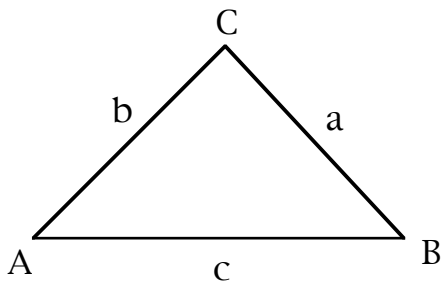
_6.1 Law of Sines

Solving a triangle: find all angles and all sides

Oblique triangle: triangles with no right angle. To solve an oblique triangle, you need to know the measure of at least one side and any two other parts of the triangle.

Sum of angles in a triangle: 180º

The Law of Sines - If A, B and C are the measures of the angles of a triangle and a, b and c are the lengths of the sides opposite these angles, then



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Applications of Law of Sines:

- can be used to solve an oblique triangle if two angles and a side are given (ASA or AAS)

- given two sides and an angle opposite one of the sides, the triangle may not exist or two triangles may exist or the triangle may be unique (SSA)

Example 1 Solve triangle (AAS) $B = 28.7°$, $C = 102.3°$ and $b = 27.4$

Example 2 Solve triangle (ASA) $A = 43^\circ,\ c = 22$ and $B = 98^\circ$

Example 3 Solve triangle (SSA) one solution.
$a = 22,\ b = 12$ and $A = 42^\circ$

Example 4 Solve triangle (SSA) No solution.
$a = 15,\ b = 25$ and $A = 85^\circ$

Example 5 Solve triangle (SSA) two solutions

$a = 12,\ b = 31$ and $A = 20.5°$.

Area of a Triangle - The area of triangle ABC is one-half the product of the lengths of any two sides and the sine of the included angle.

$$K = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$$

Example 6 Find the area of a triangular lot having two sides of lengths 90 meters and 52 meters and an included angle of 102°.

Example 7 The course for a boat race starts at point A and proceeds in the direction S 52° W to point B, then in the direction S 40° E to point C, and finally back to A, as shown in figure 6.9 (textbook page 415). Point C lies 8 kilometers directly south of point A. Approximate the total distance of the race course.