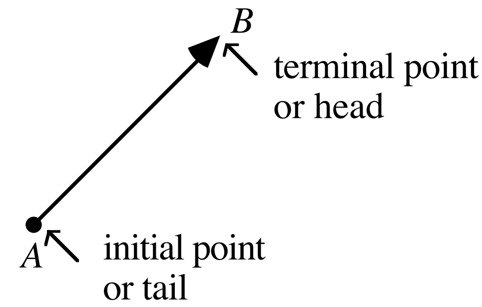
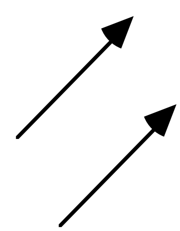
# 6.3 Vectors in the Plane

- a vector is a directed line segment
- the length of the line segment is the magnitude of the vector and the direction of the vector is measured by an angle
- the vector at the right can be denoted by $\overrightarrow{AB}$, $\overrightarrow{V}$, $\boldsymbol{AB}$ or $\boldsymbol{V}$
- the magnitude of this vector is denoted by $\|\overrightarrow{AB}\|$, $\|\overrightarrow{V}\|$, $\|\boldsymbol{AB}\|$ or $\|\boldsymbol{V}\|$

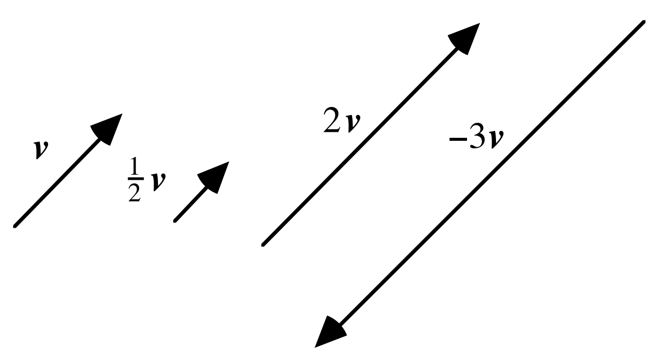



- equivalent vectors have the same magnitude and the same direction, but location is not important



- multiplying a vector by a positive real number changes the magnitude, but not the direction of the vector

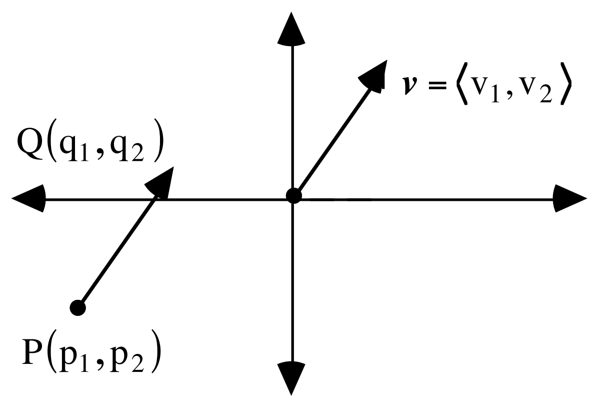
- multiplying a vector by a negative real number reverses the direction of the vector and changes its magnitude



Example 1
Let **u** be represented by the directed line segment from P = (0, 0) to Q = (3, 2), and let **v** be represented by the directed line segment from R = (1,2) to S = (4,4). Show that **u = v.**

**Component Form of a Vector**

Let $P(p_1, p_2)$ be the initial point of a vector and $Q(q_1, q_2)$ its terminal point, then an equivalent vector $v$ with initial point at the origin and terminal point $\boldsymbol{v}(v_1, v_2)$ has <u>components</u> $v_1 = q_1 - p_1$ and $v_2 = q_2 - p_2$ and can be denoted $\boldsymbol{v} = \langle v_1, v_2 \rangle$




Example 2 Find the component form and magnitude of a vector **v** with initial point P(4, –7) and terminal point Q(–1, 5).

**Fundamental Vector Operations** - If $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ are two vectors and $k$ is a scalar (real number), then

(1) $\mathbf{u} + \mathbf{v} = \langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle = \langle u_1 + v_1, u_2 + v_2 \rangle$.

(2) $k\mathbf{u} = k\langle u_1, u_2 \rangle = \langle k\,u_1, k\,u_2 \rangle$.

(3) $\|v\| = \sqrt{v_1^2 + v_2^2}$ (called magnitude or length)

Example 3 Let $\mathbf{v} = \langle -2, 5 \rangle$ and $\mathbf{w} = \langle 3, 4 \rangle$, find each of the following vectors.

a) $2\mathbf{v}$ b) $\mathbf{w} - \mathbf{v}$ c) $\mathbf{v} + 2\mathbf{w}$

**Unit Vectors**

- a <u>unit vector</u> is a vector with magnitude 1
- a unit vector in the same direction as $v$ is found by $\dfrac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$ or $\left(\dfrac{1}{\|v\|}\right)v$

Example 4 Find a unit vector in the direction of $\mathbf{v} = \langle -2, 5 \rangle$ and verify that it has magnitude of 1.

Definition of unit vectors i and j
$\text{i} = \langle 1,0 \rangle$ and $\text{j} = \langle 0,1 \rangle$

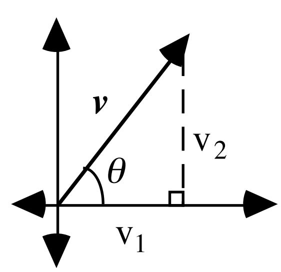
**Representation of unit vectors i and j:**

If $\text{v} = \langle \text{v}_1, \text{v}_2 \rangle$ is a vector, then $\text{v} = \text{v}_1\text{i} + \text{v}_2\text{j}$.
(This is called a linear combination of the vectors i and j.)

Example 5 Let **u** be the vector with initial point (2,–5) and terminal point (–1, 3). Write **u** as a linear combination of the standard unit vectors **i** and **j.**

Example 6 Let **u** = –3**i** + 8**j** and let **v** = 2**i** – **j**. Find 2**u** – 3**v**.

**Direction Angles**




- the angle θ is the direction angle of v measured from the positive x-axis
- by right triangles, $\tan\theta = \dfrac{\text{v}_2}{\text{v}_1}$

  (θ in the correct quadrant)

## <u>Horizontal and Vertical Components of a Vector</u>

- let $v = \langle v_1, v_2 \rangle$ be a nonzero vector
- the <u>horizontal component</u> of $v$ is $v_1 = \|v\| \cos\theta$
- the <u>vertical component</u> of $v$ is $v_2 = \|v\| \sin\theta$
- $\theta$ is the angle between the positive x-axis and $v$

<u>Note</u>: $v = \|v\| \cos\theta \, i + \|v\| \sin\theta \, j$

Example 7 Find the direction angle of each vector.

a) $\mathbf{u} = 3\mathbf{i} + 3\mathbf{j}$ **b)** $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$