6.4 Vectors and Dot Products

Dot Product

The dot product of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is:

$$u \cdot v = u_1 v_1 + u_2 v_2$$

Example 1 Find each dot product.

a) $\langle 4,5 \rangle \cdot \langle 2,3 \rangle$

b) $\langle 2,-1 \rangle \cdot \langle 1,2 \rangle$

c) $\langle 0,3 \rangle \cdot \langle 4,-2 \rangle$

Example 2 Let $\mathbf{u} = \langle -1,3 \rangle$, $\mathbf{v} = \langle 2,-4 \rangle$, and $\mathbf{w} = \langle 1,-2 \rangle$

a) $(u \cdot v)w$

b) $u \cdot 2v$

The Angle Between Two Vectors

If θ is the angle between two nonzero vectors **u** and **v**, then

$$\cos\theta = \frac{u \cdot v}{\|u\| \; \|v\|}$$ where θ is $0 \le \theta \le \pi$ and $u \cdot v$ is a dot product.

Example 3 Find the angle between $\mathbf{u} = \langle 4,3 \rangle$ and $\mathbf{v} = \langle 3,5 \rangle$

Orthogonal Vectors

The vectors **u** and **v** are orthogonal if $\mathbf{u} \cdot \mathbf{v} = 0$

Example 4 Are the vectors $\mathbf{u} = \langle 2, -3 \rangle$ and $\mathbf{v} = \langle 6, 4 \rangle$ orthogonal ?