6.5 Trigonometric Form of a Complex Number

The Complex Plane

A complex number z = a + bi is represented as the point (a, b) in the complex plane. The vertical axis is called the imaginary axis. The horizontal axis is called the real axis.

Example 1   Plot the following complex numbers.

a.) 3 + 2i   c.) -2

b.) -1 +3i   d.) –3i

Absolute Value (of a complex number)

The **absolute value** of the complex number a + bi is

$$|z| = |a + bi| = \sqrt{a^2 + b^2}.$$

Example 2   Determine the absolute value of the following complex number and plot it.   z = –2 + 5i

Trigonometric Form (Polar form) of a complex number

The complex number z = a + bi is written in polar form as $z = r(\cos\theta + i\sin\theta)$ where $a = r\cos\theta,\ b = r\sin\theta,\ r = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$. The value of r is called the **modulus** and the angle θ is called the **argument** with $0 \le \theta < 2\pi$

Example 3   Write the following in Trigonometric (polar) form.

$z = -2 - 2\sqrt{3}\, i$

Example 4 Write the following in standard (rectangular) form.

$$z = \sqrt{8}\left[\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)\right]$$

Multiplication and Division of Complex Numbers

Product and Quotient of Complex Numbers in Trigonometric (Polar) Form

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be two complex numbers.

To multiply two complex numbers, multiply moduli and add arguments.

$$z_1 z_2 = r_1 r_2\left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

To divide two complex numbers, divide moduli and subtract arguments.

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

Example 5 Find the product of the complex numbers.

$z_1 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$, $z_2 = 8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$, find $z_1 z_2$.

Example 6 Find the quotient of the complex numbers.

$z_1 = 24(\cos 300° + i\sin 300°)$, $z_2 = 8(\cos 75° + i\sin 75°)$, find $\frac{z_1}{z_2}$.

DeMoivre's Theorem

If $z = r(\cos\theta + i\sin\theta)$ is a complex number and n is a positive integer, then

$$z^n = \left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$

Example 7 Use DeMoivre's Theorem to find $\left(-1+\sqrt{3}i\right)^{12}$