Math 1113 | Sample Quiz/Test 7 | Name ______________________

Precalculus | Sections 6.3- 6.5 | Date ______________________

Directions. Show all work. Circle final answers.

1. Find the component form and the magnitude of the vector **v** if the initial point is (1,3) and terminal point is (–8,–9).

2. Find 2**u** – 3**v** given $u = \langle 2,1 \rangle$ and $v = \langle 1,3 \rangle$.

3. Find a unit vector in the direction of the vector **v** = 6**i** – 2**j**.

4. Find the **magnitude** and **direction** angle of the vector **v** = 6**i** – 6**j**.

5. Find the dot product of $u = \langle 6,1 \rangle$ and $v = \langle -2,3 \rangle$.

6. Find the angle between **u** = 3**i** + 4**j** and **v** = 0**i** – 2**j**.

7. Are the vectors **u** = $\left\langle \frac{3}{4}, \frac{-1}{4} \right\rangle$ and **v** = $\langle 5,6 \rangle$ orthogonal ?

8. Determine the absolute value of the following complex number and plot it.

$z = -4 + 4i$

9. Write the following in Trigonometric (polar) form.

$z = \sqrt{3} + i$

10. Find $z_1 z_2$. Leave answer in trig form.

$z_1 = \frac{5}{3}\left(\cos 140^{o} + i\sin 140^{o}\right)$,

$z_2 = \frac{2}{3}\left(\cos 60^{o} + i\sin 60^{o}\right)$

Answers Sample Quiz 7

1.

Initial point: $(1, 3)$

Terminal point: $(-8, -9)$

$$\mathbf{v} = \langle -8 - 1, -9 - 3 \rangle = \langle -9, -12 \rangle$$

$$\|\mathbf{v}\| = \sqrt{(-9)^2 + (-12)^2} = \sqrt{225} = 15$$

2.

$$2\mathbf{u} - 3\mathbf{v} = \langle 4, 2 \rangle - \langle 3, 9 \rangle = \langle 1, -7 \rangle$$

3.

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{6^2 + (-2)^2}}(6\mathbf{i} - 2\mathbf{j}) = \frac{1}{\sqrt{40}}(6\mathbf{i} - 2\mathbf{j})$$

$$= \frac{1}{2\sqrt{10}}(6\mathbf{i} - 2\mathbf{j}) = \frac{3}{\sqrt{10}}\mathbf{i} - \frac{1}{\sqrt{10}}\mathbf{j}$$

4.

$$\mathbf{v} = 6\mathbf{i} - 6\mathbf{j}$$

$$\|\mathbf{v}\| = \sqrt{6^2 + (-6)^2} = \sqrt{72} = 6\sqrt{2}$$

$$\tan\theta = \frac{-6}{6} = -1$$

Since $\mathbf{v}$ lies in Quadrant IV, $\theta = 315°$.

5.

$$\mathbf{u} = \langle 6, 1 \rangle, \ \mathbf{v} = \langle -2, 3 \rangle$$

$$\mathbf{u} \cdot \mathbf{v} = 6(-2) + 1(3) = -9$$

6.

$\mathbf{u} = 3\mathbf{i} + 4\mathbf{j},\ \mathbf{v} = -2\mathbf{j}$

$\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = -\dfrac{8}{(5)(2)}$

$\theta = \arccos\left(-\dfrac{4}{5}\right)$

$\theta \approx 143.13°$

7.

$\mathbf{u} = \frac{1}{4}(3\mathbf{i} - \mathbf{j}),\ \mathbf{v} = 5\mathbf{i} + 6\mathbf{j}$

$\mathbf{u} \neq k\mathbf{v} \implies$ Not parallel

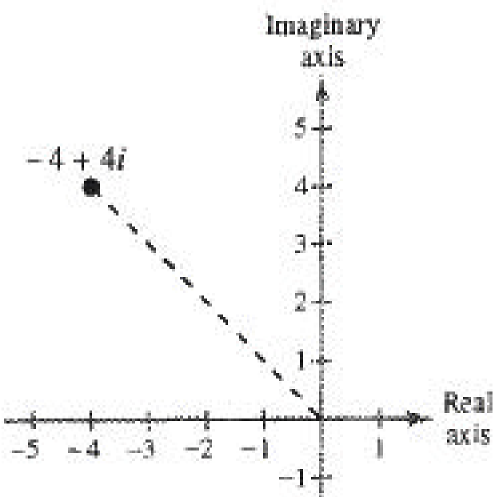
$\mathbf{u}\cdot\mathbf{v} \neq 0 \implies$ Not orthogonal

Neither

8.

$|-4 + 4i| = \sqrt{(-4)^2 + (4)^2}$

$= \sqrt{32} = 4\sqrt{2}$



9.

$z = \sqrt{3} + i$

$r = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{4} = 2$

$\tan\theta = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} \implies \theta = \dfrac{\pi}{6}$

$z = 2\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

10.

$\left[\frac{5}{3}(\cos 140° + i\sin 140°)\right]\left[\frac{2}{3}(\cos 60° + i\sin 60°)\right]$

$= \left(\frac{5}{3}\right)\left(\frac{2}{3}\right)[\cos(140° + 60°) + i\sin(140° + 60°)]$

$= \frac{10}{9}(\cos 200° + i\sin 200°)$

Formula Sheet Quiz 7

(1) $\mathbf{u} + \mathbf{v} = \langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle = \langle u_1 + v_1, u_2 + v_2 \rangle$.

(2) $k\mathbf{u} = k\langle u_1, u_2 \rangle = \langle k u_1, k u_2 \rangle$.

(3) $\|\boldsymbol{v}\| = \sqrt{v_1^2 + v_2^2}$ (called magnitude or length)

- a <u>unit vector</u> is a vector with magnitude 1
- a unit vector in the same direction as $\boldsymbol{v}$ is found by $\dfrac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$ or $\left(\dfrac{1}{\|v\|}\right)v$

$i = \langle 1,0 \rangle$ and $j = \langle 0,1 \rangle$

If $v = \langle v_1, v_2 \rangle$ is a vector, then $v = v_1 i + v_2 j$.

<u>Direction Angles</u>

- the angle θ is the <u>direction angle</u> of $\boldsymbol{v}$
- by right triangles, $\tan\theta = \dfrac{v_2}{v_1}$

  (θ in the correct quadrant)

<u>Note</u>: $\boldsymbol{v} = \|\boldsymbol{v}\| \cos\theta \, \boldsymbol{i} + \|\boldsymbol{v}\| \sin\theta \, \boldsymbol{j}$

The dot product of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ = $u \cdot v = u_1 v_1 + u_2 v_2$

If θ is the angle between two nonzero vectors **u** and **v**, then

$\cos\theta = \dfrac{u \cdot v}{\|u\| \, \|v\|}$ where θ is $0 \le \theta \le \pi$ and $u \cdot v$ is a dot product.

The **absolute value** of the complex number a + bi is $|z| = |a + bi| = \sqrt{a^2 + b^2}$.

The complex number z = a + bi is written in polar form as $z = r(\cos\theta + i\sin\theta)$ where
$a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \dfrac{b}{a}$. with $0 \le \theta < 2\pi$

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be two complex numbers.

$$z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$