Math 1111 Sample Test 3 Solutions (Chapter 6)

1) $f(x) = 9x$

$g(x) = 2x^2 + 3$

1a) $(f \circ g)(4) = f[g(4)] = f[35] = 9(35) = \boxed{315}$

$g(4) = 2(4)^2 + 3 = 2(16) + 3 = 35$

1b) $(g \circ f)(2) = g[f(2)] = g(18) = 2(18)^2 + 3 = 2(324) + 3 = \boxed{651}$

$f(2) = 9(2) = 18$

1c) $(f \circ f)(1) = f[f(1)] = f[9] = 9(9) = \boxed{81}$

$f(1) = 9(1) = 9$

1d) $(g \circ g)(0) = g[g(0)] = g[3] = 2(3)^2 + 3 = \boxed{21}$

$g(0) = 2(0)^2 + 3 = 3$

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2. $f(x) = 5x^2 - 4$

$g(x) = 6 - \frac{1}{2}x^2$

2a) $(f \circ g)(4) = f[g(4)] = f[-2] = 5(-2)^2 - 4 = 5(4) - 4 = \boxed{16}$

$g(4) = 6 - \frac{1}{2}(4)^2 = 6 - \frac{1}{2}(16) = 6 - 8 = -2$

2b) $(g \circ f)(2) = g[f(2)] = g[16] = 6 - (\frac{1}{2})(16)^2 = 6 - \frac{1}{2}(256) = \boxed{-122}$

$f(2) = 5(2)^2 - 4 = 5(4) - 4 = 16$

2c) $(f \circ f)(1) = f[f(1)] = f[1] = 5(1)^2 - 4 = 1$

$f(1) = 5(1)^2 - 4 = 1$

2d) $(g \circ g)(0) = g[g(0)] = g[6] = 6 - \frac{1}{2}(6)^2 = 6 - 18 = -12$

$g(0) = 6 - \frac{1}{2}(0)^2 = 6$

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3. $f(x) = 2\sqrt{x}$

$g(x) = 7x$

3a) $(f \circ g)(4) = f[g(4)] = f[28] = 2\sqrt{28} = 2\sqrt{4}\sqrt{7} = 4\sqrt{7}$

$g(4) = 7(4) = 28$

3b) $(g \circ f)(2) = g[f(2)] = g[2\sqrt{2}] = 7(2\sqrt{2}) = 14\sqrt{2}$

$f(2) = 2\sqrt{2}$

3c) $(f \circ f)(1) = f[f(1)] = f[2] = 2\sqrt{2}$

$f(1) = 2\sqrt{1} = 2$

3d) $(g \circ g)(0) = g[g(0)] = g[0] = 2\sqrt{0} = 0$

$g(0) = 7(0) = 0$

4) $f(x) = 4x$

$g(x) = \frac{x}{4}$

$(f \circ g)(x) = f[g(x)] = 4\left(\frac{x}{4}\right) = x$ ✓

$(g \circ f)(x) = g[f(x)] = \frac{4x}{4} = x$ ✓

$(f \circ g)(x) = (g \circ f)(x)$? yes

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5) $f(x) = 4x - 2$

$g(x) = \frac{1}{4}(x+2)$

$(f \circ g)(x) = f[g(x)] = 4\left(\frac{1}{4}(x+2)\right) - 2$

$= x + 2 - 2 = x$ ✓

$(g \circ f)(x) = g[f(x)] = \frac{1}{4}((4x-2)+2)$

$= \frac{1}{4}(4x) = x$ ✓

$(f \circ g)(x) = (g \circ f)(x)$? yes

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6) HLT no

7) $\{(0,-5),(-13,5),(9,2),(-5,-3),(5,-11)\}$

inverse

$\{(-5,0),(5,-13),(2,9),(-3,-5),(-11,5)\}$

Domain of inverse

$\{-5,5,2,-3,-11\}$

Range of inverse

$\{0,-13,9,-5,5\}$

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8) $f(x) = x^3 - 9$

$g(x) = \sqrt[3]{x+9}$

8a) $f(g(x)) = \left(\sqrt[3]{x+9}\right)^3 - 9$

$= x+9-9 = (x)$

None

8b) $g(f(x)) = \sqrt[3]{(x^3-9)+9} = \sqrt[3]{x^3} = (x)$

none

8c) yes inverse

9) choice A

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10) $f(x) = 5x$

10a) inverse $y = 5x$

$\frac{x}{5} = \frac{5y}{5}$

$y = \frac{x}{5}$

$f^{-1}(x) = \frac{x}{5}$

10b) look at graph

$D = (-\infty, \infty)$ or $\mathbb{R}$

$R = (-\infty, \infty)$ or $\mathbb{R}$

10c) look at graph

$D = (-\infty, \infty)$ or $\mathbb{R}$

$R = (-\infty, \infty)$ $\mathbb{R}$

10d) choice (D)

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11) $f(x) = 5x + 1$

11a) find $f^{-1}(x)$

$y = 5x + 1$

$x = 5y + 1$

$-1 \quad -1$

$\frac{x-1}{5} = \frac{5y}{5}$

$y = \frac{x-1}{5}$

$f^{-1}(x) = \frac{x-1}{5}$

11b) $D = \mathbb{R}$ $R = \mathbb{R}$ (look at graph)

11c $D = \mathbb{R}$ $R = \mathbb{R}$ (look at graph)

11d choice (A)

12) $y = 2^{-(x-1)}$ Choice (G)

$y = 2^{-x+1}$ or 2^{1-x} shift right 1, reflects over y-axis

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13) $f(x) = 5^{-x} + 5$ reflects over y-axis

graph choice (D) shift up 5 Choice (B)

$D = (-\infty, \infty)$ or $\mathbb{R}$

$R = (5, \infty)$

H.A $y = 5$

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14) $f(x) = 6 + e^{-x}$ reflects over y axis

shift up 6

choice (A)

$D = (-\infty, \infty)$ or $\mathbb{R}$

$R = (6, \infty)$

H.A. $y = 6$

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15) $\left(\frac{3}{2}\right)^x = \left(\frac{27}{8}\right)$ solve

$\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^3$

$x = 3$

16) $4^{3x+1} = 64$

$4^{3x+1} = 4^3$

$3x + 1 = 3$

$3x = 2$

$x = \frac{2}{3}$

17) $8.2 = a^6$

$\log_a 8.2 = 6$

18) $\log_2 8 = x$

$2^x = 8$

19) $\log_7(49) = x$

$7^x = 49$

$7^x = 7^2$

$x = 2$

20) $f(x) = \ln(x-1)$

Domain of $\ln x = (0, \infty)$

So $\ln(x-1)$ shift right 1

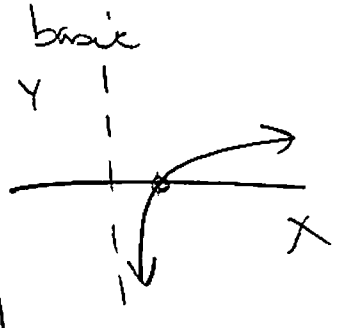
$D = (1, \infty)$ also can look at graph!

21) $\dfrac{\ln \frac{13}{10}}{0.03}$ type: $\ln(13/10)/.03 = 8.745$

22) shift right 1, reflect over y-axis

$\log_4(x-1) = \log_{\frac{1}{4}} x + 1$

Choice (H) or $\log_4(1-x)$



23) $\log_3 x = 4$

$3^4 = x$

$x = 81$ check? (yes)

24) $\log_2(4x+9) = 3$

$2^3 = 4x+9$

$4x+9 = 8$

$4x = -1$

$x = -\frac{1}{4}$ check? yes

25) $e^{8x} = 7$

$\ln_e 7 = 8x$

$\frac{\ln_e 7}{8} = \frac{8x}{8}$

$x = \frac{\ln 7}{8}$ or .243

26) $\log_6 24 - \log_6 4$

$= \log_6 \frac{24}{4} = \log_6 6$

$= 1$

27) $\ln 5.5 = \ln \frac{11}{2}$

$= \ln 11 - \ln 2 =$ t-s

28) $\log_2 8x$

$= \log_2 8 + \log_2 x$

$= \log_2 2^3 + \log_2 x$

$= 3\log_2 2 + \log_2 x$

$= 3 + \log_2 x$

29) $5\log_3 u + 9\log_3 v$

$= \log_3 u^5 + \log_3 v^9$

$= \boxed{\log_3 u^5 \cdot v^9}$

30) $\log_3(x^2-64) - 9\log_3(x+8)$

$= \log_3(x^2-64) - \log_3(x+8)^9$

$= \log_3 \frac{x^2-64}{(x+8)^9} = \log_3 \frac{(x+8)(x-8)}{(x+8)^9}$

$= \boxed{\log_3 \frac{(x-8)}{(x+8)^8}}$

31) $\log_7 42 = \frac{\log_{10}(42)}{\log_{10}(7)}$

$= \boxed{1.921}$

32) $\log_{\frac{1}{5}} 8 = \frac{\log_{10}(8)}{\log_{10}(\frac{1}{5})}$

$= \boxed{-1.292}$

33) $\log_2(5x) = 4$

$2^4 = 5x$

$\frac{5x}{5} = \frac{16}{5}$

$\boxed{x = \frac{16}{5}}$ check? yes

34) $2\log_3(x) = -\log_3 9$

$\log_3 x^2 = \log_3 9^{-1}$

$x^2 = 9^{-1}$

$x^2 = \frac{1}{9}$

$x = \pm\frac{1}{3}$ check?

$\boxed{x = +\frac{1}{3}}$ ~~$x = -\frac{1}{3}$~~

35) $\log_5(x+131) + \log_5(x+11) = 4$

$\log_5(x+131)(x+11) = 4$

$\log_5(x^2 + 11x + 131x + 1441) = 4$

$\log_5(x^2 + 142x + 1441) = 4$

$5^4 = x^2 + 142x + 1441$

$625 = x^2 + 142x + 1441$

$x^2 + 142x + 816 = 0$

$(x+136)(x+6) = 0$

$x = -136$ ~~$x = -136$~~ $x = -6$

check?

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36) $3^x = 7$

$\ln 3^x = \ln 7$

$\frac{x \ln 3}{\ln 3} = \frac{\ln 7}{\ln 3}$

$x = \frac{\ln 7}{\ln 3}$ or 1.771

37) $6^{1-9x} = 7^{x}$

$\ln 6^{1-9x} = \ln 7^{x}$

$(1-9x)(\ln 6) = x(\ln 7)$

$\ln 6 - 9x \ln 6 = x \ln 7$

$\ln 6 = x \ln 7 + 9x \ln 6$

$$\frac{\ln 6}{\ln 7 + 9\ln 6} = \frac{x(\ln 7 + 9\ln 6)}{\ln 7 + 9\ln 6}$$

$$x = \frac{\ln 6}{\ln 7 + 9\ln 6}$$ or .099