

§6.7 Compound Interest

Simple Interest Formula

If a principle of P dollars is borrowed for a period of t years at a per annum interest rate r , expressed as a decimal, the interest I charged is $I = Prt$

$$P = 1000 \quad r = .09 \quad t = 5 \quad I =$$

Formulas for Compound Interest:

After t years, the balance A in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas:

1. For n compoundings per year:

$$A = P \left(1 + \frac{r}{n} \right)^{(n \cdot t)}$$

2. For continuous compounding:

$$A = Pe^{(r \cdot t)}$$

Example (future value): A total of \$12,000 is invested at an annual interest rate of 9%. Find the balance after 5 years if it is compounded:

a) quarterly. $n=4$

$$\begin{aligned} A &= P \left(1 + \frac{r}{n} \right)^{(n \cdot t)} \\ &= 12000 \left(1 + \frac{.09}{4} \right)^{(4 \cdot 5)} \\ &= \underline{18726.11} \end{aligned}$$

b) continuously.

$$\begin{aligned} A &= Pe^{(r \cdot t)} \\ &= 12000e^{(.09 \cdot 5)} \\ &= \underline{18919.75} \end{aligned}$$

A =
P =
r =
t =
n =

Compound Interest (rate of interest):

Example : What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years?

$$A = P \left(1 + \frac{r}{n}\right)^{(n \cdot t)}$$
$$\frac{2P}{P} = \frac{P \left(1 + \frac{r}{1}\right)^{(1 \cdot 5)}}{P}$$
$$2 = (1+r)^5$$
$$\sqrt[5]{2} = \sqrt[5]{(1+r)^5}$$
$$\sqrt[5]{2} = 1+r$$
$$r = \sqrt[5]{2} - 1$$

Continuous Compounding:

Example : How long will it take for the money in an account that is compounded continuously at 5% to double? Triple?

$$A = P e^{(r \cdot t)}$$
$$\frac{2P}{P} = \frac{P e^{(r \cdot t)}}{P}$$
$$2 = e^{(.05t)}$$
$$\ln 2 = \ln e^{.05t}$$
$$\frac{.05t}{.05} = \frac{\ln 2}{.05}$$
$$t = 13.96$$