§4.1 Polynomial Functions and Models

A polynomial function is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and n is a nonnegative integer.

Determine which of the following are polynomial functions. For those that are, state the degree; for those that are not, tell why not.

(a) $f(x) = 2 - 3x^4$

(b) $g(x) = \sqrt{x}$

(c) $h(x) = \dfrac{x^2 - 2}{x^3 - 1}$

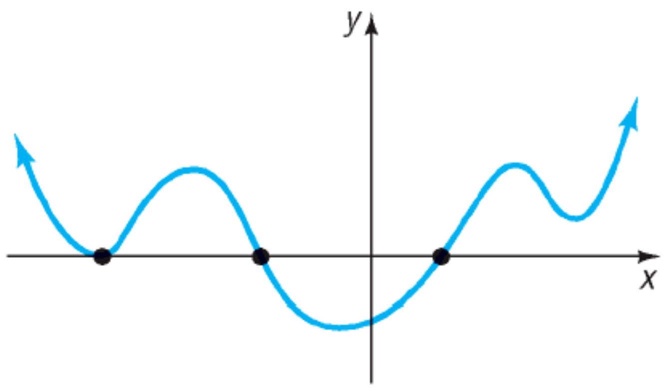
(d) $F(x) = 0$

(e) $G(x) = 8$

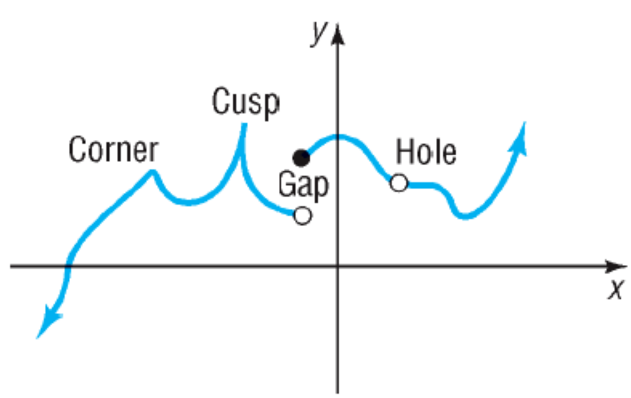
(f) $H(x) = -2x^3(x - 1)^2$

Summary of the Properties of the Graphs of Polynomial Functions

Degree	Form	Name	Graph
No degree	$f(x) = 0$	Zero function	The x-axis
0	$f(x) = a_0, \quad a_0 \neq 0$	Constant function	Horizontal line with y-intercept a_0
1	$f(x) = a_1x + a_0, \quad a_1 \neq 0$	Linear function	Nonvertical, nonhorizontal line with slope a_1 and y-intercept a_0
2	$f(x) = a_2x^2 + a_1x + a_0, \quad a_2 \neq 0$	Quadratic function	Parabola: Graph opens up if $a_2 > 0$; graph opens down if $a_2 < 0$



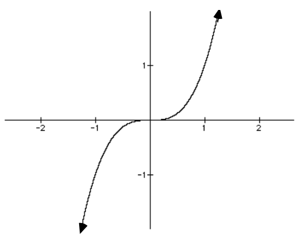
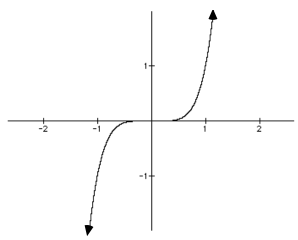
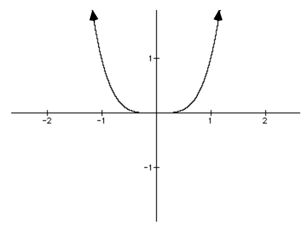
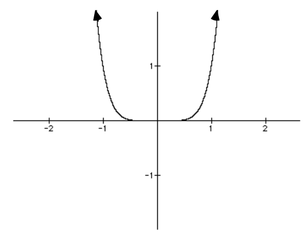
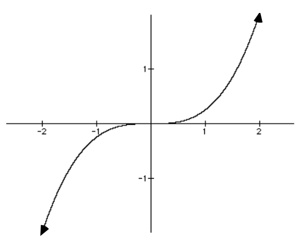
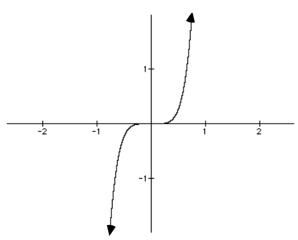
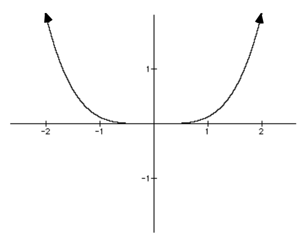
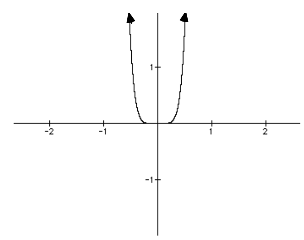
(a) Graph of a polynomial function: smooth, continuous



(b) Cannot be the graph of a polynomial function

Polynomials are **continuous** (no breaks in the graph) and **smooth** (no sharp angles, only rounded curves)

Graphing Functions of the Form: $P(x) = ax^n$

$P(x) = x^3$	$P(x) = x^5$	$P(x) = x^4$	$P(x) = x^6$
			
			
$P(x) = \frac{1}{3}x^3$	$P(x) = 8x^5$	$P(x) = \frac{1}{8}x^4$	$P(x) = 9x^6$

Note: The graph of $y = x^n$ is similar to the graph of

$\begin{cases} y = x^2 \text{ if } n \text{ is even} \\ y = x^3 \text{ if } n \text{ is odd} \end{cases}$, except that the greater n is, the flatter the graph is on [-1, 1] and the steeper it is on $(-\infty,-1)\cup(1,\infty)$.

Examining Vertical and Horizontal Translations (Shifts):

Example 1: Graph

a.) $y = -(x + 2)^4 + 6$

b.) $y = -3 - (x - 1)^3$

Finding a polynomial from its Zeros:

Example Find a polynomial of degree 3 whose zeros are -4, -2, and 3.

Identifying Zeros and Their Multiplicities

For the polynomial, list all zeros and their multiplicities.

$$f(x) = -2(x-2)(x+1)^3(x-3)^4$$

If r Is a Zero of Even Multiplicity

Sign of $f(x)$ does not change from one side of r to the other side of r. Graph **touches** x-axis at r.

If r Is a Zero of Odd Multiplicity

Sign of $f(x)$ changes from one side of r to the other side of r. Graph **crosses** x-axis at r.

Theorem

Turning Points

If f is a polynomial function of degree n, then f has at most $n - 1$ turning points.

If the graph of a polynomial function f has $n - 1$ turning points, the degree of f is at least n.

Example Graphing a Polynomial using x-intercepts

For the polynomial: $f(x) = x^2(x - 2)$

(a) Find the x- and y-intercepts of the graph of f.

(b) Use the x-intercepts to find the intervals on which the graph of f is above the x-axis and the intervals on which the graph of f is below the x-axis.

(c) Locate other points on the graph and connect all the points plotted with a smooth, continuous curve.

Interval	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
Number Chosen	-1	1	3
Value of *f*	$f(-1) = -3$	$f(1) = -1$	$f(3) = 9$
Location of Graph	Below x-axis	Below x-axis	Above x-axis
Point on Graph	$(-1, -3)$	$(1, -1)$	$(3, 9)$