§4.2 Properties of Rational Functions

Rational Function - a function of the form

$$f(x)=\frac{p(x)}{q(x)}$$ where p(x) and q(x) are polynomials with $q(x)\neq 0$.

Find the Domain of a Rational Function

The domain is the set of all real numbers where the denominator $\neq 0$.

(a) The domain of $R(x)=\dfrac{2x^2-4}{x+5}$ is the set of all real numbers x except -5; that is, $\{x \mid x \neq -5\}$.

(b) The domain of $R(x)=\dfrac{1}{x^2-4}$ is the set of all real numbers x except -2 and 2, that is, $\{x \mid x \neq -2, x \neq 2\}$.

(c) The domain of $R(x)=\dfrac{x^3}{x^2+1}$ is the set of all real numbers.

(d) The domain of $R(x)=\dfrac{-x^2+2}{3}$ is the set of all real numbers.

(e) The domain of $R(x)=\dfrac{x^2-1}{x-1}$ is the set of all real numbers x except 1, that is, $\{x \mid x \neq 1\}$. ◀

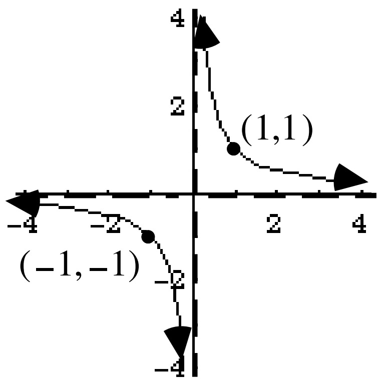
It is important to observe that the functions

$$R(x)=\frac{x^2-1}{x-1} \quad \text{and} \quad f(x)=x+1$$

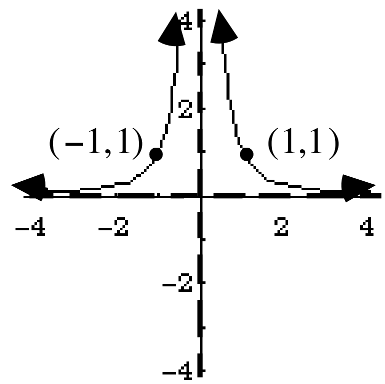
are not equal, since the domain of R is $\{x \mid x \neq 1\}$ and the domain of f is the set of all real numbers.

The graphs of rational functions approach (get closer and closer to) lines called **<u>asymptotes</u>**.

Basic Graphs (memorize these)



$y = \frac{1}{x}$



$y = \frac{1}{x^2}$

Example 1 Use stretching/shrinking, reflecting and shifting rules to graph the following.

a.) $f(x) = \frac{1}{x+3} - 1$

b.) $f(x) = \frac{1}{(x-2)^2} + 1$

To Find the Asymptotes of a Rational Function:

(1) Vertical Asymptotes – When the function is in lowest terms you can find any vertical asymptotes by setting the denominator equal to 0 and solving for x to get the equation $\boxed{\mathbf{x = a}}$.

(2) Horizontal Asymptotes

Rule 1: If the numerator has lower degree than the denominator, the horizontal asymptote is $\boxed{\mathbf{y = 0}}$.

Rule 2: If the numerator and denominator have the same degree and a_n is the leading coefficient of the numerator and b_n is the leading coefficient of the denominator, the horizontal asymptote is $\boxed{y = \frac{a_n}{b_n}}$.

Rule 3: If the numerator has higher degree than the denominator, there is **no horizontal asymptote**.

(3) Slant Asymptotes - If the numerator is of degree exactly one more than the denominator, there is an slant asymptote. To find it, divide the numerator by the denominator and disregard any remainder. The equation of the slant asymptote is the result of setting y = to the quotient.

Example 2 Give the equations of the vertical, horizontal and/or slant asymptotes of the rational function.

a.) $f(x) = \dfrac{3x}{(x+1)(x-2)}$

b.) $f(x) = \dfrac{(x-5)(x-2)}{x^2+9}$

c.) $f(x) = \dfrac{2x^2+3}{x-4}$

d.) $f(x) = \dfrac{2(3x-1)(x+4)}{(x+2)(5x-3)}$

Example 3 Find the x-intercepts and y-intercept of the rational function.

a.) $f(x) = \dfrac{3x}{(x+1)(x-2)}$

b.) $f(x) = \dfrac{(x-5)(x-2)}{x^2+9}$