§4.4 Polynomial and Rational Inequalities

Steps for Solving Polynomial and Rational Inequalities

STEP 1: Write the inequality so that a polynomial or rational expression f is on the left side and zero is on the right side in one of the following forms:

$$f(x) > 0 \qquad f(x) \geq 0 \qquad f(x) < 0 \qquad f(x) \leq 0$$

For rational expressions, be sure that the left side is written as a single quotient.

STEP 2: Determine the numbers at which the expression f on the left side equals zero and, if the expression is rational, the numbers at which the expression f on the left side is undefined.

STEP 3: Use the numbers found in Step 2 to separate the real number line into intervals.

STEP 4: Select a number in each interval and evaluate f at the number.

(a) If the value of f is positive, then $f(x) > 0$ for all numbers x in the interval.

(b) If the value of f is negative, then $f(x) < 0$ for all numbers x in the interval.

If the inequality is not strict, include the solutions of $f(x) = 0$ in the solution set.

Rational Inequalities:

Note:– **NEVER** multiply both sides of an inequality by a variable expression!!
- – You cannot lose the denominator in quotients.
- – Always remember the restriction that the denominator cannot be zero.

Examples   Solve.
a.) $x^4 \le 4x^2$

b.) $x^4 > x$

c.) $\dfrac{(x+3)(2-x)}{(x-1)^2} > 0$

d.) $\dfrac{4x+5}{x+2} \geq 3$