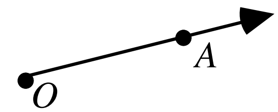
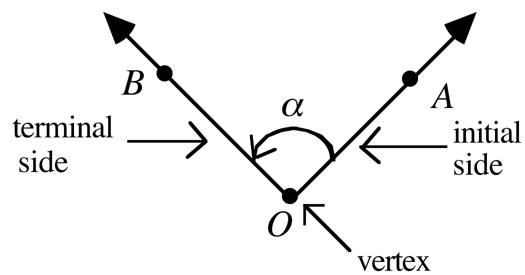
§6.1 Angles and Their Measure

So what does trigonometry mean ? **measurement of triangles!**

- a ray starts at a point and extends indefinitely

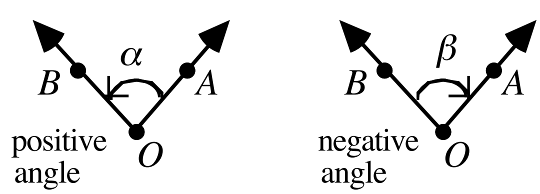


- an angle occurs when a ray is rotated about its endpoint
- the starting position of the ray is the initial side of the angle
- the position of the ray after rotation is the terminal side of the angle
- the meeting point of the two rays is the vertex of the angle



$\angle\alpha = \angle O = \angle AOB$

- a positive angle is formed by a counter-clockwise rotation
- a negative angle is formed by a clockwise rotation



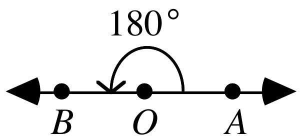
- coterminal angles have the same initial and terminal sides.

Degree Measure

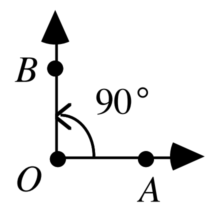
• an angle formed by rotating a ray $\frac{1}{360}$ of a complete revolution has a measure of 1 degree (1°)

• angles are often classified by their measures

(1) a straight angle has a measure of 180°

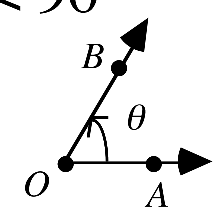


(2) a right angle has a measure of 90°



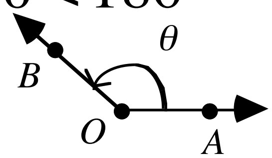
(3) an acute angle has a measure

$0° < \theta < 90°$

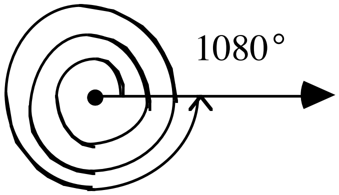


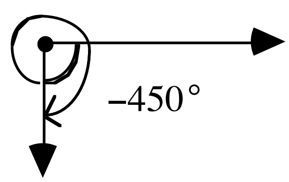
(4) an obtuse angle has a measure

$90° < \theta < 180°$



• angles larger than 360° or smaller than −360° can be measured by considering more than one rotation

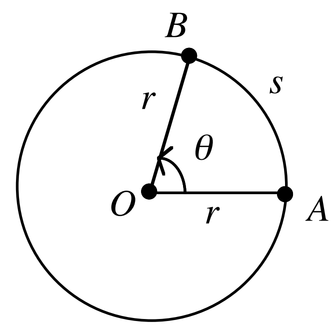




Draw an Angle: (discuss standard position)

a) 45° b) −90° c) 225° d) 405°

Radian Measure



- consider a circle of radius r with two radii OA and OB
- the angle θ formed by these two radii is a central angle
- the arc AB is the part of the circle between A and B and its length is S
- the arc AB subtends the angle θ

- the measure of the central angle subtended by an arc of length r on a circle with radius r is one radian

- the radian measure of the central angle subtended by an arc of length s on a circle of radius r is $\theta = \frac{s}{r}$ or $s = r\theta$

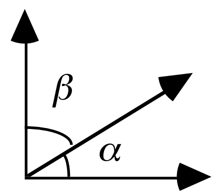
- given a circle of radius r, the radian measure of the central angle subtended by the circumference of the circle is $\theta = \frac{2\pi r}{r} = 2\pi$ while in degrees $\theta = 360°$

- thus, $360° = 2\pi$ radians and $180° = \pi$ radians

Example: Find the Arc Length of a Circle

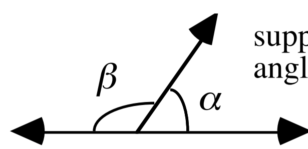
Find the length of an arc of a circle of radius 2 meters subtended by a central angle of 0.25 rdian.

- two **nonnegative** angles α and β are <u>complementary angles</u> if $\alpha + \beta = 90°$
- in this case, α is the <u>complement</u> of β and vice versa



complementary angles

- two **nonnegative** angles α and β are <u>supplementary angles</u> if $\alpha + \beta = 180°$
- in this case, α is the <u>supplement</u> of β and vice versa



supplementary angles

Example Find the coterminal angles for the following angles.

a) $390°$ b) $-225°$

Example 2 Find the complement and supplement angles for the following angles.

a) $42°$ b) $145°$

Radian-Degree Conversion Factors

- to change radians to degrees, multiply the number of radians by $\frac{180°}{\pi}$
- to change degrees to radians, multiply the number of degrees by $\frac{\pi}{180°}$

Example Convert from Degrees to Radians.

a) $60°$ b) $-45°$ c) $107°$

Example Convert from Radians to Degrees

a) $\frac{\pi}{6}$ b) $-\frac{3\pi}{2}$ c) 3 radians

DMS System (Degree, Minute, Second)

1 minute $(1') = \left(\frac{1}{60}\right)^\circ \Rightarrow 60' = 1^\circ$

1 second $(1'') = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^\circ \Rightarrow 60'' = 1'$ and $3600'' = 1^\circ$

Example Convert $50^\circ 6' 21''$ to decimal degree measure to the nearest thousandth.

Example Convert 21.256° to DMS.

Note: You **MUST** memorize all degree to radian conversions of the selected angles listed below and know their positions on a circle measured from the positive x-axis.

Degrees	Radians
0	0
30	$\pi/6$
45	$\pi/4$
60	$\pi/3$
90	$\pi/2$
120	$2\pi/3$
135	$3\pi/4$
150	$5\pi/6$
180	π
210	$7\pi/6$
225	$5\pi/4$
240	$4\pi/3$
270	$3\pi/2$
300	$5\pi/3$
315	$7\pi/4$
330	$11\pi/6$
360	2π