

## §6.3 Properties of the Trigonometric Functions

### Domain(Range) and Period of Sine and Cosine

Domain of the Sine and Cosine is all real numbers:

$$-\infty < t < \infty$$

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Range of Sine	$-1 \leq y \leq 1$
	$-1 \leq \sin t \leq 1$

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Range of Cosine	$-1 \leq x \leq 1$
	$-1 \leq \cos t \leq 1$

### Definition of a periodic function

A function  $f$  is **periodic** if there exists a positive real number  $c$  such that

$$f(t + c) = f(t)$$

for all  $t$  in the domain of  $f$ . The smallest number  $c$  for which  $f$  is periodic is called the **period** of  $f$ .

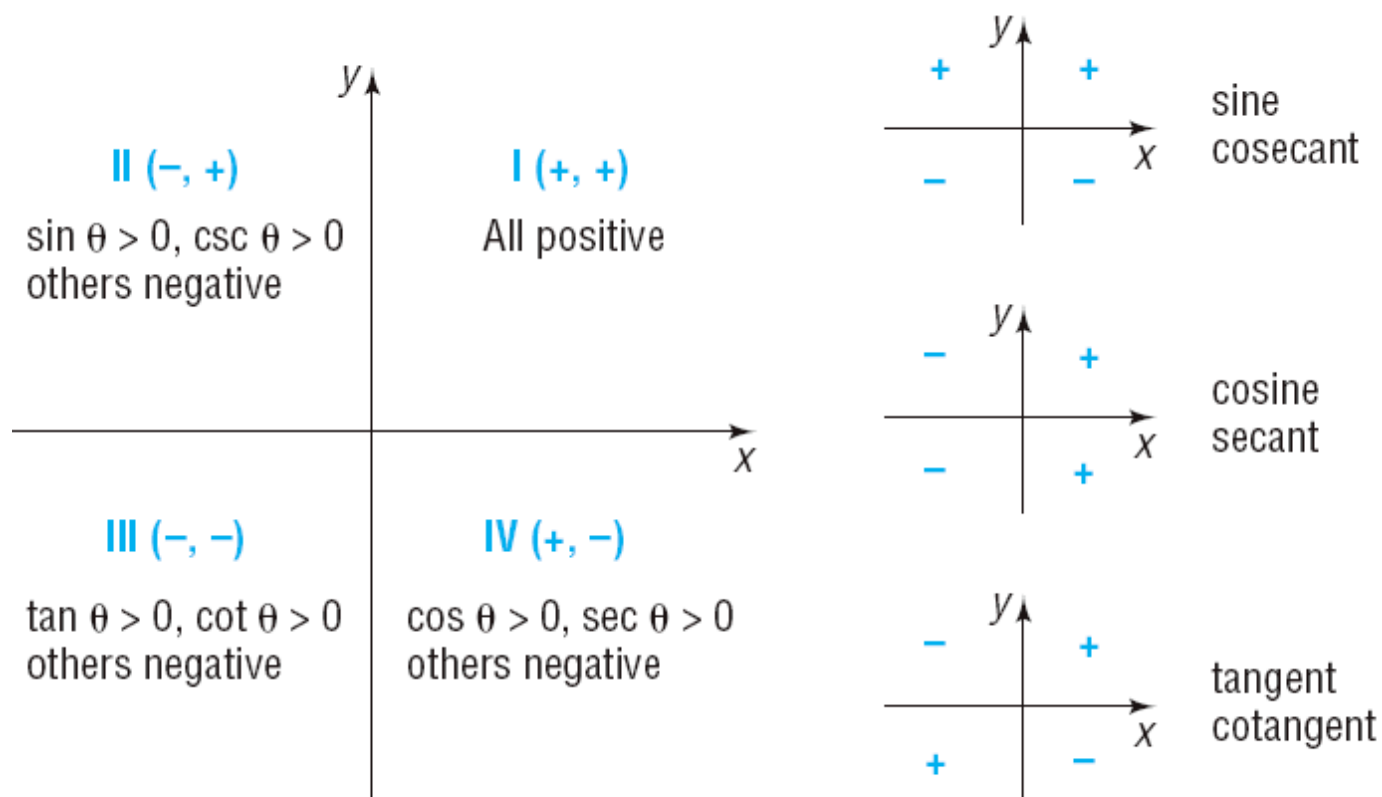
Examples: Find

a)  $\sin \frac{17\pi}{4}$

b)  $\cos 5\pi$

c)  $\tan \frac{5\pi}{4}$

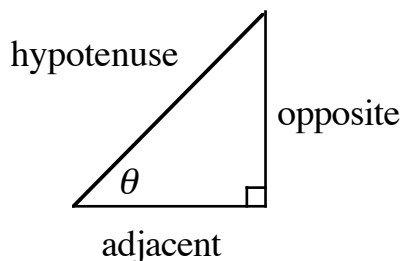
# Determine the signs of the Trig Functions in a Given Quadrant



If  $\sin \theta < 0$  and  $\cos \theta < 0$ , name the quadrant in which the angle lies.

# Find the Values of the Trig Functions Using Fundamental Identities

## Right Triangle Trigonometry



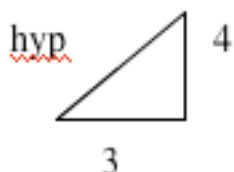
## Right Triangle Definitions of Trigonometric Functions

Let  $\theta$  be an acute angle of a right triangle. Then:

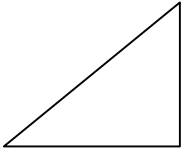
$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos \theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec \theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot \theta = \frac{\text{adj}}{\text{opp}}$$

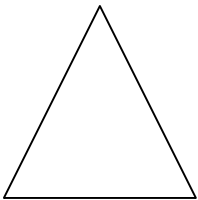
Example Evaluate the six trig functions for this triangle:



Example : Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ , and  $\tan 45^\circ$  using right triangle trigonometry.



Example : Find the values of  $\sin 30^\circ$ ,  $\cos 30^\circ$ ,  $\sin 60^\circ$ , and  $\cos 60^\circ$  using right triangle trigonometry.



# Trigonometric Identities

## Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta} \quad \cos \theta = \frac{1}{\sec \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

## Quotient or Ratio Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

## Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta$$
$$1 + \cot^2 \theta = \csc^2 \theta$$

Example: Find the exact values of the remaining four trig functions of  $\theta$  using identities.

$$\sin \theta = \frac{\sqrt{5}}{5} \quad \text{and} \quad \cos \theta = \frac{2\sqrt{5}}{5}$$

Example : Given that  $\sin \theta = \frac{1}{3}$  and  $\cos \theta < 0$ , find the exact value of each of the remaining five trig functions of  $\theta$ .

Example : Given that  $\tan \theta = \frac{1}{2}$  and  $\sin \theta < 0$ , find the exact value of each of the remaining five trig functions of  $\theta$ .

## Even and Odd Trigonometric Functions

The cosine and secant functions are **even**.

$$\cos(-t) = \cos t \qquad \sec(-t) = \sec t$$

The sine, cosecant, tangent, and cotangent functions are **odd**.

$$\sin(-t) = -\sin(t) \qquad \csc(-t) = -\csc(t)$$

$$\tan(-t) = -\tan(t) \qquad \cot(-t) = -\cot(t)$$

Find the exact value of:

a)  $\sin(-45^\circ)$       b)  $\cos(-\pi)$       c)  $\cot\left(-\frac{3\pi}{2}\right)$