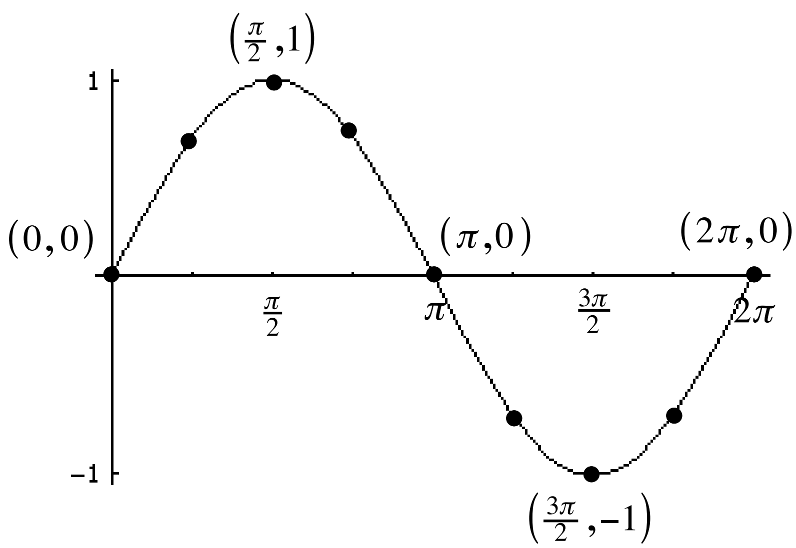
§6.4 Graphs of Sine and Cosine Functions

Graph of $y = \sin x$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
y = sin x									



- since the domain of $y = \sin x$ is all real numbers, the graph repeats infinitely to the left and the right

- one <u>period</u> (or <u>cycle</u>) of the graph is on $[0,2\pi]$

Graphing trigonometric functions on TI calculator

MODE all choices on left should be highlighted, radians

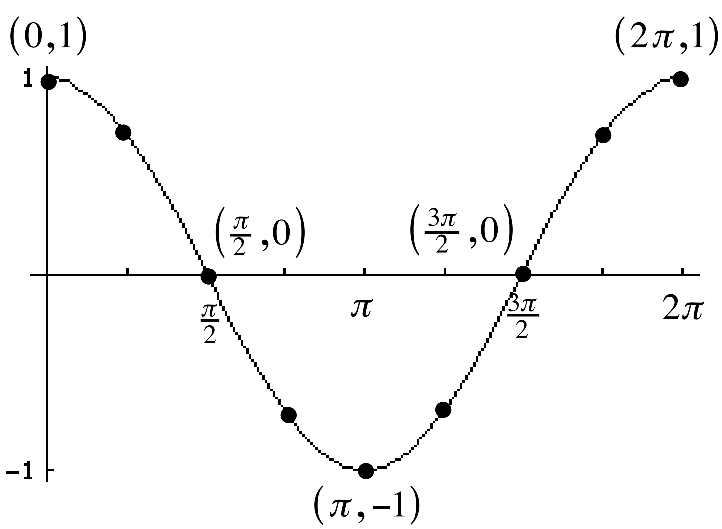
WINDOW	xmin	-2π	(endpoint left)
	xmax	2π	(endpoint right)
	xscl	$\pi/2$	(tick marks)
	ymin	-2	(Amplitude low)
	ymax	2	(Amplitude high)
	yscl	1	(ignore)

Example: Graph y = sin x on your calculator. Draw the axes and label properly.

Example: Sketch the graph of y = 2 sin x on the interval $[-\pi, 4\pi]$. Remember key points.

Graph of $y = \cos x$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
y = cos x									



- since the domain of y = cos x is all real numbers, the graph repeats infinitely to the left and the right

- one <u>period</u> (or <u>cycle</u>) of the graph is on $[0, 2\pi]$

Amplitude

Compare the graph $y = \sin x$ to each of the following: <u>(Vertical Shrinking and Stretching)</u>

Ex 1. $y = 2\sin x$

2. $y = -3\sin x$

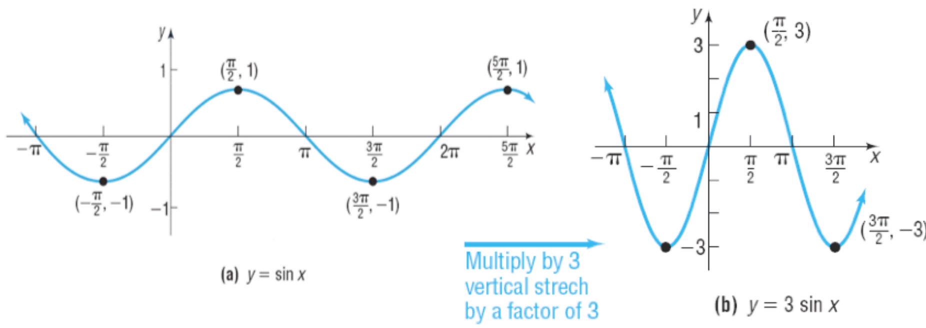
3. $y = \frac{1}{2}\sin x$

Period

EXAMPLE

Graphing Functions of the Form $y = A\sin(\omega x)$ Using Transformations

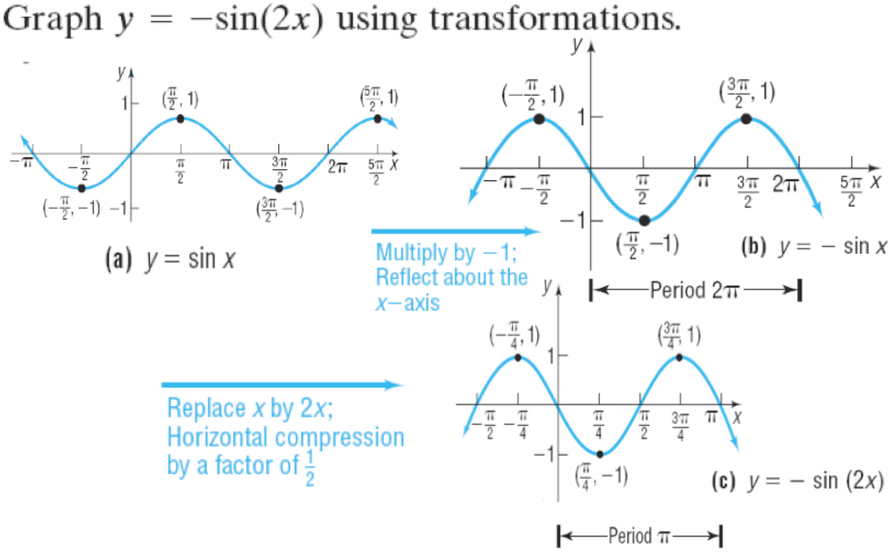
Graph $y = 3\sin x$ using transformations.



EXAMPLE

Graphing Functions of the Form $y = A\sin(\omega x)$ Using Transformations

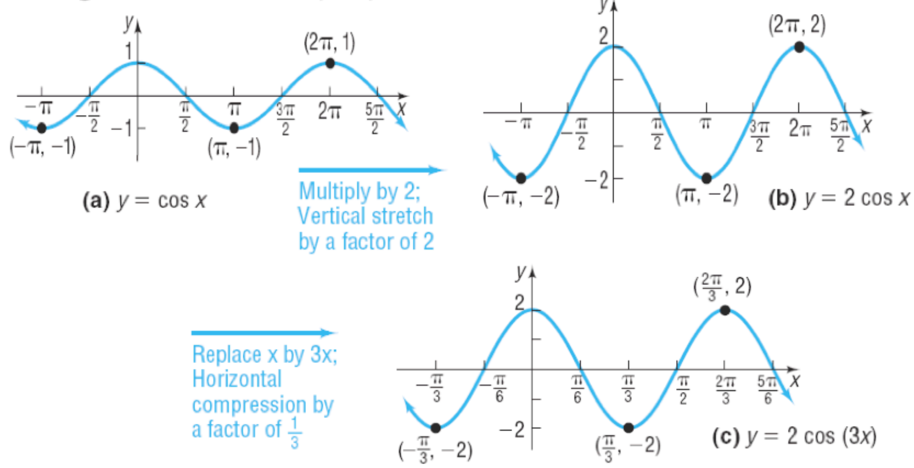
Graph $y = -\sin(2x)$ using transformations.



EXAMPLE

Graphing Functions of the Form $y = A\cos(\omega x)$ Using Transformations

Graph $y = 2\cos(3x)$ using transformations.



(a) $y = \cos x$ **(b)** $y = 2\cos x$ **(c)** $y = 2\cos(3x)$

Formulas for General Form $y = a\sin(bx - c) + d$ **and** $y = a\cos(bx - c) + d$

amplitude = $|a|$

period (of sine and cosine) = $\frac{2\pi}{b}$

tick marks = $\frac{\text{period}}{4}$

endpoints Solve: $bx - c = 0$ $bx - c = 2\pi$

vertical shift = d

Example: Horizontal Translation

Sketch the graph of $y = \frac{1}{2}\sin\left(x - \frac{\pi}{3}\right)$

Example: Horizontal Translation

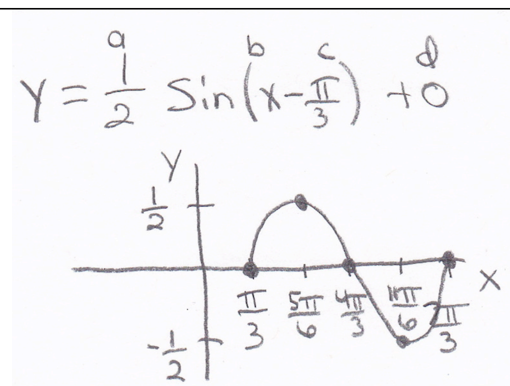
Sketch the graph of $y = -3\cos(2\pi x + 4\pi)$

Example: Vertical Translation

Sketch the graph of $y = 2 + 3\cos(2x)$

Example: $y = \frac{1}{2}\sin\left(x - \frac{\pi}{3}\right)$ (Remember APTEV)

Formulas for General Form $y = a\sin(bx - c) + d$ and $y = a\cos(bx - c) + d$

amplitude = $\lvert a \rvert = \left\lvert \frac{1}{2} \right\rvert = \frac{1}{2}$ period (of sine and cosine) = $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$ tick marks = $\frac{\text{period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$	tick mark calculations: (1) $\frac{\pi}{3}$ (2) $\frac{\pi}{3} + \frac{\pi}{2} = \frac{5\pi}{6}$ (3) $\frac{5\pi}{6} + \frac{\pi}{2} = \frac{8\pi}{6} = \frac{4\pi}{3}$ (4) $\frac{4\pi}{3} + \frac{\pi}{2} = \frac{11\pi}{6}$ (5) $\frac{11\pi}{6} + \frac{\pi}{2} = \frac{7\pi}{3}$
endpoints: $bx - c = 0$; $x - \frac{\pi}{3} = 0$; $x = \frac{\pi}{3}$ (starts) Solve: $bx - c = 2\pi$; $x - \frac{\pi}{3} = 2\pi$; $x = \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3}$ (ends)	

vertical shift = d = none

Example: $y = 3\cos(2x) + 2$ (Remember APTEV)

Formulas for General Form $y = a\sin(bx - c) + d$ and $y = a\cos(bx - c) + d$

amplitude = $|a| = |3| = 3$

---

period (of sine and cosine) =

$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

---

tick marks = $\frac{\text{period}}{4} = \frac{\pi}{4}$

tick mark calculations:

(1) 0

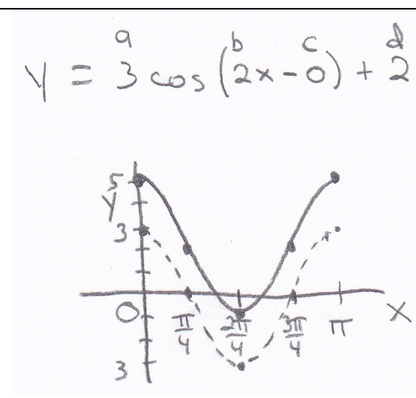
(2) $0 + \frac{\pi}{4} = \frac{\pi}{4}$

(3) $\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

(4) $\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

(5) $\frac{3\pi}{4} + \frac{\pi}{4} = \frac{4\pi}{4} = \pi$

endpoints	Solve:
$bx - c = 0$	$bx - c = 2\pi$
$2x - 0 = 0$	$2x - 0 = 2\pi$
$x = 0$	$x = \pi$
(starts)	(ends)



---

vertical shift = d = 2

Example : $y = -3\cos(2\pi x + 4\pi)$ (Remember APTEV)

Formulas for General Form $y = a\sin(bx - c) + d$ and $y = a\cos(bx - c) + d$

amplitude = $|a| = |-3| = 3$

---

period (of sine and cosine) =

$$\frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$$

---

tick marks = $\frac{\text{period}}{4} = \frac{1}{4}$

tick mark calculations:

(1) -2

(2) $-2 + \frac{1}{4} = \frac{-7}{4}$

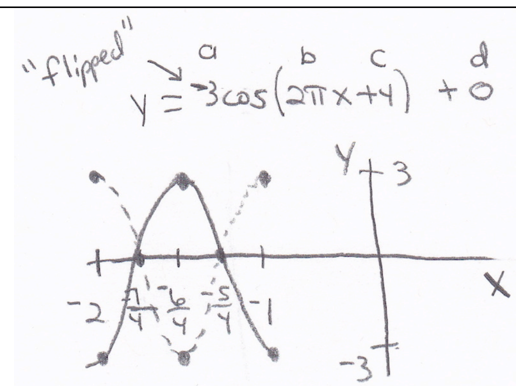
(3) $\frac{-7}{4} + \frac{1}{4} = \frac{-6}{4} = \frac{-3}{2}$

(4) $\frac{-3}{2} + \frac{1}{4} = \frac{-5}{4}$

(5) $\frac{-5}{4} + \frac{1}{4} = -1$

endpoints

bx − c = 0	Solve: bx − c = 2π
$2\pi x + 4\pi = 0$	$2\pi x + 4\pi = 2\pi$
$2\pi x = -4\pi$	$2\pi x = -2\pi$
$x = -2$	$x = -1$
(starts)	(ends)



---

vertical shift = d = none