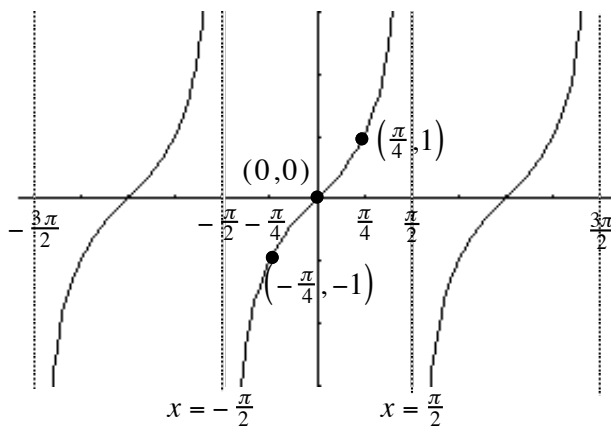


§6.5 Graphs of the Other Trigonometric Functions

Graph of $y = \tan x$

| | | | | | |
|----------|------------------|------------------|-----|-----------------|-----------------|
| x | $-\frac{\pi}{2}$ | $-\frac{\pi}{4}$ | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ |
| $\tan x$ | undefined | -1 | 0 | 1 | undefined |



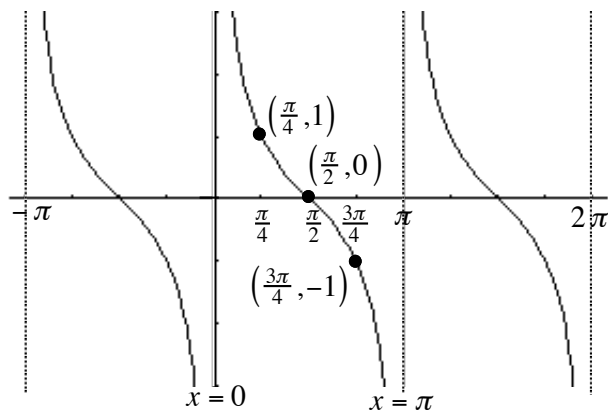
since the domain of $y = \tan x$ is all real numbers except $\frac{(2n+1)\pi}{2}$, the graph repeats infinitely to the left and the right

one period (or cycle) of the graph is on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Example 1: Graph a) $y = \tan \frac{x}{2}$ b) $y = -3 \tan 2x$

Graph of $y = \cot x$

| | | | | | |
|----------|-----------|-----------------|-----------------|------------------|-----------|
| x | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | π |
| $\cot x$ | undefined | 1 | 0 | -1 | undefined |



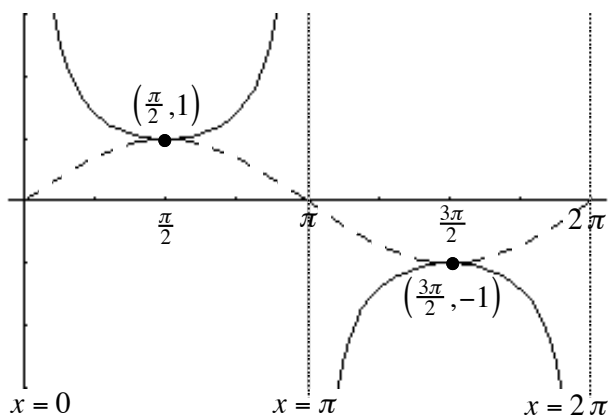
since the domain of $y = \cot x$ is all real numbers except $n\pi$, the graph repeats infinitely to the left and the right

one period (or cycle) of the graph is on $[0, \pi]$

Example 2: Graph $y = 2 \cot \frac{x}{3}$

Graph of $y = \csc(x)$

| x | 0 | $\frac{\pi}{2}$ | π | $\frac{3\pi}{2}$ | 2π |
|--------------|-----------|-----------------|-----------|------------------|-----------|
| $y = \csc x$ | undefined | 1 | undefined | -1 | undefined |

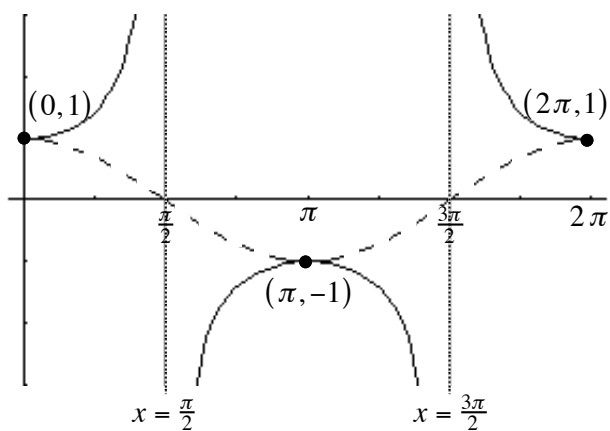


since the domain of $y = \csc x$ is all real numbers except $n\pi$, the graph repeats infinitely to the left and the right

one period (or cycle) of the graph is on $[0, 2\pi]$

Graph of $y = \sec(x)$

| | | | | | |
|--------------|-----|-----------------|-------|------------------|--------|
| x | 0 | $\frac{\pi}{2}$ | π | $\frac{3\pi}{2}$ | 2π |
| $y = \sec x$ | 1 | undefined | -1 | undefined | 1 |



since the domain of $y = \sec x$ is all real numbers except $\frac{(2n+1)\pi}{2}$, the graph repeats infinitely to the left and the right

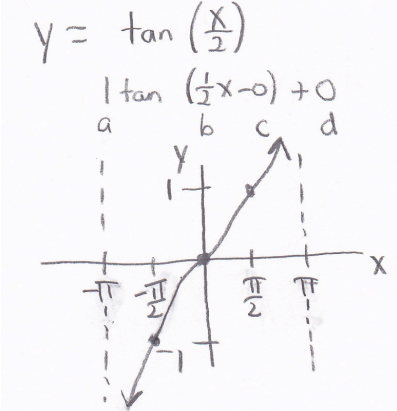
one period (or cycle) of the graph is on $[0, 2\pi]$

Example 3: Graph a) $y = 2 \csc\left(x + \frac{\pi}{4}\right)$ b) $y = \sec(2x)$

Example: $y = \tan\left(\frac{x}{2}\right)$

(Remember APTEV)

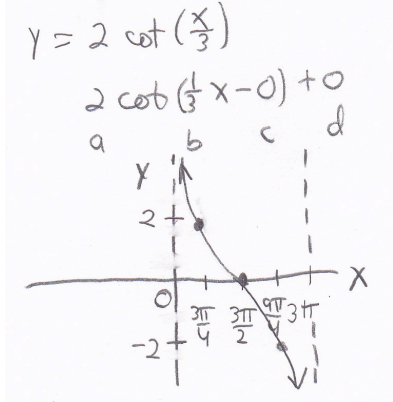
Formulas for General Form $y = a \tan(bx - c) + d$

| | |
|---|--|
| <p>amplitude = none</p> <p>period (of tan and cot) =</p> $\frac{\pi}{b} = \frac{\pi}{1/2} = 2\pi$ <p>tick marks = $\frac{\text{period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$</p> | <p>tick mark calculations:</p> <p>(1) $-\pi$ (2) $-\pi + \frac{\pi}{2} = -\frac{\pi}{2}$</p> <p>(3) $\frac{-\pi}{2} + \frac{\pi}{2} = 0$ (4) $0 + \frac{\pi}{2} = \frac{\pi}{2}$</p> <p>(5) $\frac{\pi}{2} + \frac{\pi}{2} = \pi$</p> |
| <p>endpoints Solve:</p> $bx - c = \frac{-\pi}{2} \quad bx - c = \frac{\pi}{2}$ $\frac{x}{2} = \frac{-\pi}{2} \quad \frac{x}{2} = \frac{\pi}{2}$ $x = -\pi \quad x = \pi$ <p>(starts) (ends)</p> |  |
| <p>vertical shift = none</p> | |

Example: $y = 2 \cot\left(\frac{x}{3}\right)$

(Remember APTEV)

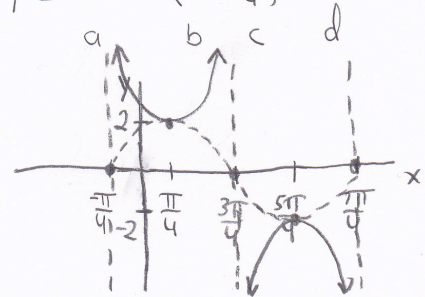
Formulas for General Form $y = a \cot(bx - c) + d$

| | |
|--|--|
| <p>amplitude = none</p> <p>period (of tan and cot) =</p> $\frac{\pi}{b} = \frac{\pi}{1/3} = 3\pi$ <p>tick marks = $\frac{\text{period}}{4} = \frac{3\pi}{4}$</p> | <p>tick mark calculations:</p> <p>(1) 0</p> <p>(2) $0 + \frac{3\pi}{4} = \frac{3\pi}{4}$</p> <p>(3) $\frac{3\pi}{4} + \frac{3\pi}{4} = \frac{3\pi}{2}$</p> <p>(4) $\frac{3\pi}{2} + \frac{3\pi}{4} = \frac{9\pi}{4}$</p> <p>(5) $\frac{9\pi}{4} + \frac{3\pi}{4} = 3\pi$</p> |
| <p>endpoints Solve:</p> <p>$bx - c = 0$ $bx - c = \pi$</p> <p>$\frac{x}{3} = 0$ $\frac{x}{3} = \pi$</p> <p>$x = 0$ $x = 3\pi$</p> <p>(starts) (ends)</p> |  |
| <p>vertical shift = none</p> | |

Example: $y = 2 \csc\left(x + \frac{\pi}{4}\right)$

(Remember APTEV)

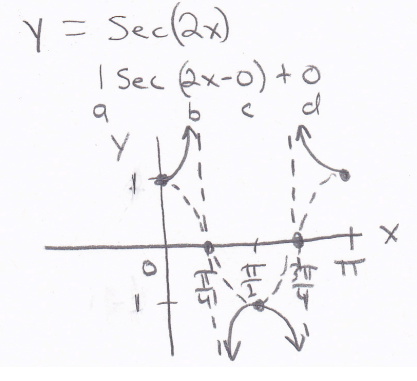
Formulas for General Form $y = a \sin(bx - c) + d$ **and** $y = a \cos(bx - c) + d$

| | |
|---|--|
| <p>amplitude = $a = 2 = 2$</p> <p>period (of sine and cosine) =</p> $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$ <p>tick marks = $\frac{\text{period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$</p> | <p>tick mark calculations:</p> <ol style="list-style-type: none"> (1) $\frac{-\pi}{4}$ (2) $\frac{-\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$ (3) $\frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$ (4) $\frac{3\pi}{4} + \frac{\pi}{2} = \frac{5\pi}{4}$ (5) $\frac{5\pi}{4} + \frac{\pi}{2} = \frac{7\pi}{4}$ |
| <p>endpoints Solve:</p> <p>$bx - c = 0$ $bx - c = 2\pi$</p> <p>$x + \frac{\pi}{4} = 0$ $x + \frac{\pi}{4} = 2\pi$</p> <p>$x = \frac{-\pi}{4}$ $x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$</p> <p>(starts) (ends)</p> | <p>$y = 2 \csc\left(x + \frac{\pi}{4}\right) + 0$</p>  |
| <p>vertical shift = $d = \text{none}$</p> | |

Example: $y = \sec(2x)$

(Remember APTEV)

Formulas for General Form $y = a \sin(bx - c) + d$ **and** $y = a \cos(bx - c) + d$

| | |
|--|---|
| <p>amplitude = $a = 1 = 1$</p> <p>period (of sine and cosine) =</p> $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ <p>tick marks = $\frac{\text{period}}{4} = \frac{\pi}{4}$</p> | <p>tick mark calculations:</p> <p>(1) 0</p> <p>(2) $0 + \frac{\pi}{4} = \frac{\pi}{4}$</p> <p>(3) $\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$</p> <p>(4) $\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$</p> <p>(5) $\frac{3\pi}{4} + \frac{\pi}{4} = \pi$</p> |
| <p>endpoints Solve:</p> <p>$bx - c = 0$ $bx - c = 2\pi$</p> <p>$2x = 0$ $2x = 2\pi$</p> <p>$x = 0$ $x = \pi$</p> <p>(starts) (ends)</p> |  |
| <p>vertical shift = $d = \text{none}$</p> | |