

In Problems 19–98, establish each identity.

- 19.** $\csc \theta \cdot \cos \theta = \cot \theta$ **20.** $\sec \theta \cdot \sin \theta = \tan \theta$ **21.** $1 + \tan^2(-\theta) = \sec^2 \theta$
- 22.** $1 + \cot^2(-\theta) = \csc^2 \theta$ **23.** $\cos \theta(\tan \theta + \cot \theta) = \csc \theta$ **24.** $\sin \theta(\cot \theta + \tan \theta) = \sec \theta$
- 25.** $\tan u \cot u - \cos^2 u = \sin^2 u$ **26.** $\sin u \csc u - \cos^2 u = \sin^2 u$ **27.** $(\sec \theta - 1)(\sec \theta + 1) = \tan^2 \theta$
- 28.** $(\csc \theta - 1)(\csc \theta + 1) = \cot^2 \theta$ **29.** $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$ **30.** $(\csc \theta + \cot \theta)(\csc \theta - \cot \theta) = 1$
- 31.** $\cos^2 \theta(1 + \tan^2 \theta) = 1$ **32.** $(1 - \cos^2 \theta)(1 + \cot^2 \theta) = 1$ **33.** $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$
- 34.** $\tan^2 \theta \cos^2 \theta + \cot^2 \theta \sin^2 \theta = 1$ **35.** $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$ **36.** $\csc^4 \theta - \csc^2 \theta = \cot^4 \theta + \cot^2 \theta$
- 37.** $\sec u - \tan u = \frac{\cos u}{1 + \sin u}$ **38.** $\csc u - \cot u = \frac{\sin u}{1 + \cos u}$ **39.** $3 \sin^2 \theta + 4 \cos^2 \theta = 3 + \cos^2 \theta$
- 40.** $9 \sec^2 \theta - 5 \tan^2 \theta = 5 + 4 \sec^2 \theta$ **41.** $1 - \frac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta$ **42.** $1 - \frac{\sin^2 \theta}{1 - \cos \theta} = -\cos \theta$
- 43.** $\frac{1 + \tan v}{1 - \tan v} = \frac{\cot v + 1}{\cot v - 1}$ **44.** $\frac{\csc v - 1}{\csc v + 1} = \frac{1 - \sin v}{1 + \sin v}$ **45.** $\frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} = 2 \tan \theta$
- 46.** $\frac{\csc \theta - 1}{\cot \theta} = \frac{\cot \theta}{\csc \theta + 1}$ **47.** $\frac{1 + \sin \theta}{1 - \sin \theta} = \frac{\csc \theta + 1}{\csc \theta - 1}$ **48.** $\frac{\cos \theta + 1}{\cos \theta - 1} = \frac{1 + \sec \theta}{1 - \sec \theta}$
- 49.** $\frac{1 - \sin v}{\cos v} + \frac{\cos v}{1 - \sin v} = 2 \sec v$ **50.** $\frac{\cos v}{1 + \sin v} + \frac{1 + \sin v}{\cos v} = 2 \sec v$ **51.** $\frac{\sin \theta}{\sin \theta - \cos \theta} = \frac{1}{1 - \cot \theta}$
- 52.** $1 - \frac{\sin^2 \theta}{1 + \cos \theta} = \cos \theta$ **53.** $\frac{1 - \sin \theta}{1 + \sin \theta} = (\sec \theta - \tan \theta)^2$ **54.** $\frac{1 - \cos \theta}{1 + \cos \theta} = (\csc \theta - \cot \theta)^2$
- 55.** $\frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta$ **56.** $\frac{\cot \theta}{1 - \tan \theta} + \frac{\tan \theta}{1 - \cot \theta} = 1 + \tan \theta + \cot \theta$



57. $\tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \sec \theta$
58. $\frac{\sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{\tan \theta}{1 - \tan^2 \theta}$
59. $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \tan \theta + \sec \theta$
60. $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{\sin \theta + 1}{\cos \theta}$
61. $\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = \frac{\sin^2 \theta - \cos^2 \theta}{1 - \tan^2 \theta}$
62. $\frac{\sec \theta - \cos \theta}{\sec \theta + \cos \theta} = \frac{\sin^2 \theta}{1 + \cos^2 \theta}$
63. $\frac{\tan u - \cot u}{\tan u + \cot u} + 1 = 2 \sin^2 u$
64. $\frac{\tan u - \cot u}{\tan u + \cot u} + 2 \cos^2 u = 1$
65. $\frac{\sec \theta + \tan \theta}{\cot \theta + \cos \theta} = \tan \theta \sec \theta$
66. $\frac{\sec \theta}{1 + \sec \theta} = \frac{1 - \cos \theta}{\sin^2 \theta}$
67. $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} + 1 = 2 \cos^2 \theta$
68. $\frac{1 - \cot^2 \theta}{1 + \cot^2 \theta} + 2 \cos^2 \theta = 1$
69. $\frac{\sec \theta - \csc \theta}{\sec \theta \csc \theta} = \sin \theta - \cos \theta$
70. $\frac{\sin^2 \theta - \tan \theta}{\cos^2 \theta - \cot \theta} = \tan^2 \theta$
71. $\sec \theta - \cos \theta = \sin \theta \tan \theta$
72. $\tan \theta + \cot \theta = \sec \theta \csc \theta$
73. $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 2 \sec^2 \theta$
74. $\frac{1 + \sin \theta}{1 - \sin \theta} - \frac{1 - \sin \theta}{1 + \sin \theta} = 4 \tan \theta \sec \theta$
75. $\frac{\sec \theta}{1 - \sin \theta} = \frac{1 + \sin \theta}{\cos^3 \theta}$
76. $\frac{1 + \sin \theta}{1 - \sin \theta} = (\sec \theta + \tan \theta)^2$
77. $\frac{(\sec v - \tan v)^2 + 1}{\csc v(\sec v - \tan v)} = 2 \tan v$
78. $\frac{\sec^2 v - \tan^2 v + \tan v}{\sec v} = \sin v + \cos v$
79. $\frac{\sin \theta + \cos \theta}{\cos \theta} - \frac{\sin \theta - \cos \theta}{\sin \theta} = \sec \theta \csc \theta$
80. $\frac{\sin \theta + \cos \theta}{\sin \theta} - \frac{\cos \theta - \sin \theta}{\cos \theta} = \sec \theta \csc \theta$
81. $\frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} = 1 - \sin \theta \cos \theta$
82. $\frac{\sin^3 \theta + \cos^3 \theta}{1 - 2 \cos^2 \theta} = \frac{\sec \theta - \sin \theta}{\tan \theta - 1}$
83. $\frac{\cos^2 \theta - \sin^2 \theta}{1 - \tan^2 \theta} = \cos^2 \theta$