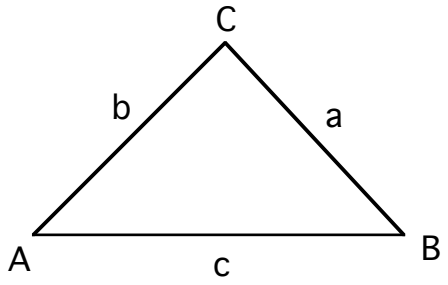
§ 8.2 Law of Sines

Solving a triangle: find all angles and all sides

Oblique triangle: triangles with no right angle. To solve an oblique triangle, you need to know the measure of at least one side and any two other parts of the triangle.

Sum of angles in a triangle: 180°

The Law of Sines - If A, B and C are the measures of the angles of a triangle and a, b and c are the lengths of the sides opposite these angles, then



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Applications of Law of Sines:

- can be used to solve an oblique triangle if two angles and a side are given (ASA or AAS)

- given two sides and an angle opposite one of the sides, the triangle may not exist or two triangles may exist or the triangle may be unique (SSA)

Example 1 Solve triangle (AAS)

$A = 40^\circ$, $B = 60^\circ$ and $a = 4$

Example 2 Solve triangle (ASA)

$A = 35^\circ$, $c = 5$ and $B = 15^\circ$

Example 3 Solve triangle (SSA) one solution.

$a = 3$, $b = 2$ and $A = 40^\circ$

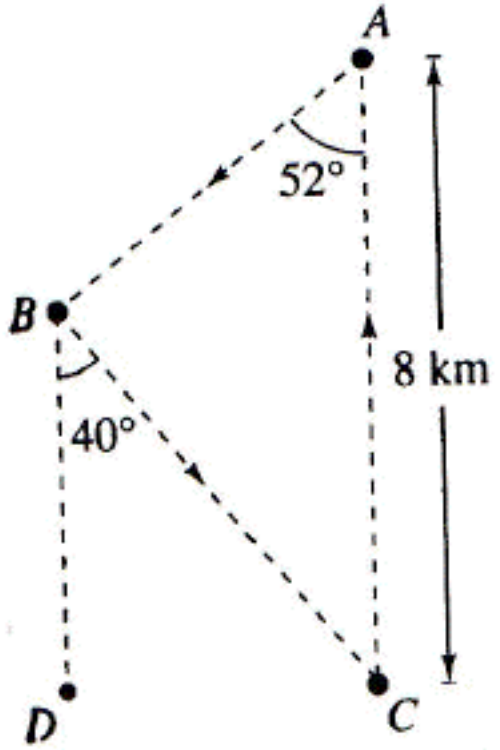
Example 4 Solve triangle (SSA) No solution.

$a = 2,\ c = 1$ and $C = 50^\circ$

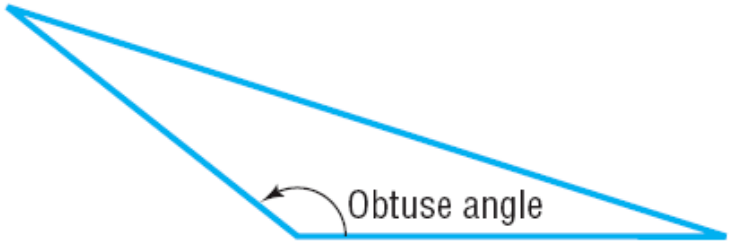
Example 5 Solve triangle (SSA) two solutions

$a = 6,\ b = 8$ and $A = 35^\circ$.

Example 6 The course for a boat race starts at point A and proceeds in the direction S 52° W to point B, then in the direction S 40° E to point C, and finally back to A, as shown in figure. Point C lies 8 kilometers directly south of point A. Approximate the total distance of the race course.

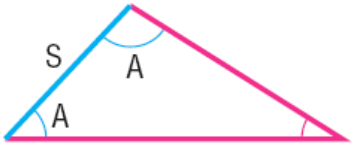


(a) All angles are acute



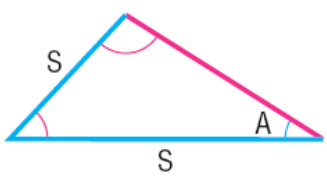
(b) Two acute angles and one obtuse angle

CASE 1: One side and two angles are known (ASA or SAA).
CASE 2: Two sides and the angle opposite one of them are known (SSA).
CASE 3: Two sides and the included angle are known (SAS).
CASE 4: Three sides are known (SSS).

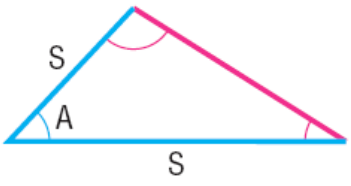


Case 1: ASA

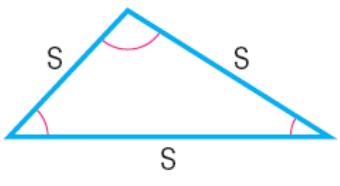
Case 1: SAA



Case 2: SSA



Case 3: SAS



Case 4: SSS

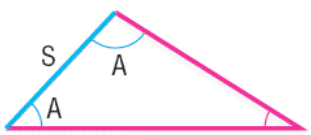
The **Law of Sines** is used to solve triangles for which Case 1 or 2 holds.

Theorem

Law of Sines

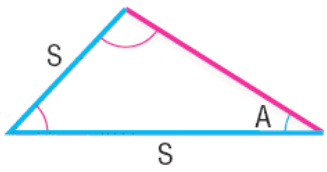
For a triangle with sides a, b, c and opposite angles A, B, C, respectively,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad (1)$$



Case 1: ASA

Case 1: SAA



Case 2: SSA

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \frac{\sin A}{a} = \frac{\sin C}{c} \qquad \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$A + B + C = 180^\circ$$