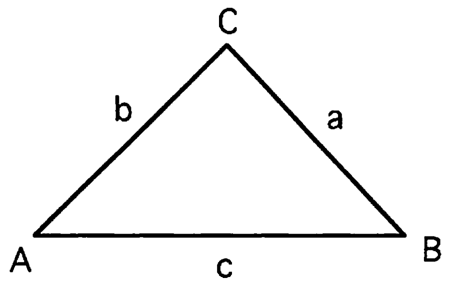
§ 8.2 Law of Sines

Solving a triangle: find all angles and all sides

Oblique triangle: triangles with no right angle. To solve an oblique triangle, you need to know the measure of at least one side and any two other parts of the triangle.

Sum of angles in a triangle: 180°

The Law of Sines - If A, B and C are the measures of the angles of a triangle and a, b and c are the lengths of the sides opposite these angles, then



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

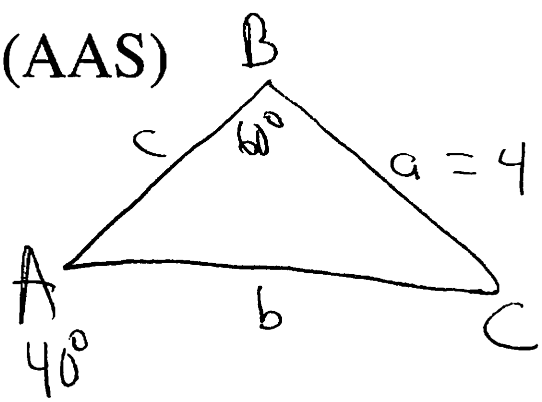
Applications of Law of Sines:

- can be used to solve an oblique triangle if two angles and a side are given (ASA or AAS)

- given two sides and an angle opposite one of the sides, the triangle may not exist or two triangles may exist or the triangle may be unique (SSA)

Example 1 Solve triangle (AAS)

A = 40°, B = 60° and a = 4



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 40^\circ}{4} = \frac{\sin 60^\circ}{b}$$

$$\frac{\sin 40^\circ\, b}{\sin 40^\circ} = \frac{\sin 60^\circ\, 4}{\sin 40^\circ}$$

$$b = 5.39$$

$a = 4$ $A = 40^\circ$

$b = 5.39$ $B = 60^\circ$

$c = 6.13$ $C = 180^\circ - 40^\circ - 60^\circ = 80^\circ$

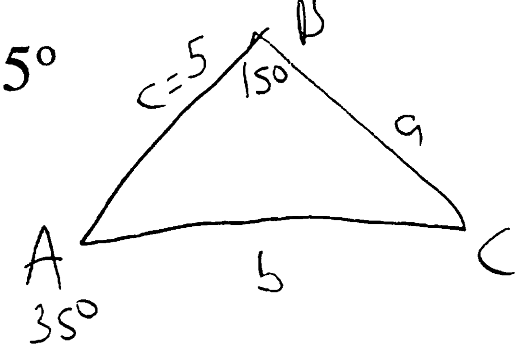
$$\frac{\sin 40^\circ}{4} = \frac{\sin C}{c}$$

$$\frac{\sin 40^\circ}{4} = \frac{\sin 80^\circ}{c}$$

$$c = \frac{\sin 80^\circ (4)}{\sin 40} = 6.13$$

Example 2 Solve triangle (ASA)

A = 35°, c = 5 and B = 15°



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 35^\circ}{a} = \frac{\sin 130^\circ}{5}$$

$$\Rightarrow \quad a = \frac{5\sin 35^\circ}{\sin 130^\circ} = 3.74$$

$a = 3.74$ $\quad A = 35^\circ$

$b =$ $\quad B = 15^\circ$

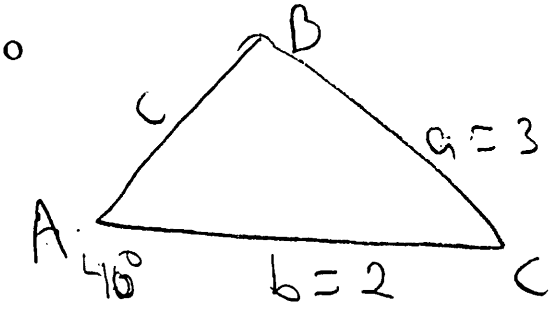
$c = 5$ $\quad C = 180 - 35^\circ - 15^\circ = 130^\circ$

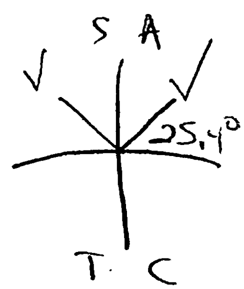
$$\frac{\sin 130^\circ}{5} = \frac{\sin 15^\circ}{b}$$

$$b = \frac{5\sin 15^\circ}{\sin 130^\circ} = 1.69$$

Example 3 Solve triangle (SSA) one solution.

a = 3, b = 2 and A = 40°





$$\frac{\sin A}{a} = \frac{\sin B^\circ}{b}$$

$$\frac{\sin 40^\circ}{3} = \frac{\sin B^\circ}{2}$$

$$\frac{2\sin 40^\circ}{3} = \frac{3\sin B^\circ}{3}$$

$$\sin B^\circ = 0.43$$

$$B^\circ = \sin^{-1}(.43) = 25.4^\circ$$

$B = 25.4^\circ$ $\quad b = 2$

$A = 40^\circ$ $\quad a = 3$

$C = 114.6^\circ$ $\quad c = 4.24$

$180 - 25.4^\circ$

$= \cancel{154.6^\circ}$

$B_1 = 25.4^\circ$

$\cancel{B_2 = 154.6^\circ}$

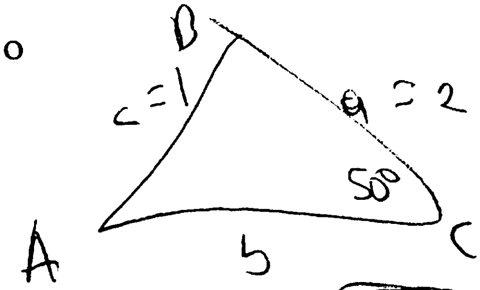
$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 114.6}{c} = \frac{\sin 40^\circ}{3}$$

$$c = \frac{3\sin 114.6^\circ}{\sin 40^\circ} = 4.24$$

Example 4 Solve triangle (SSA) No solution.

a = 2, c = 1 and C = 50°



$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 50^\circ}{1} = \frac{\sin A}{2}$$

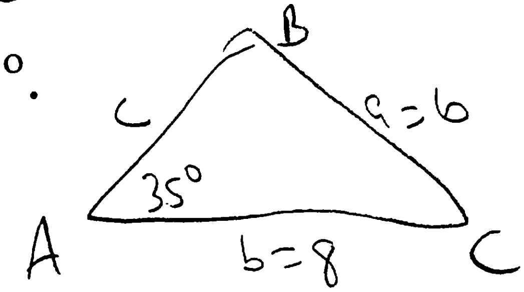
no soln

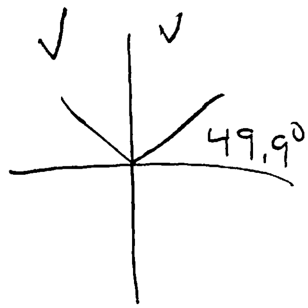
$$\sin A^\circ = 2\sin 50^\circ$$

$$A^\circ = \sin^{-1}(2\sin 50^\circ) = \sin^{-1}(1.53)$$

Example 5 Solve triangle (SSA) two solutions

a = 6, b = 8 and A = 35°.





$$\frac{\sin A}{a} = \frac{\sin B^\circ}{b}$$

$$\frac{\sin 35^\circ}{6} = \frac{\sin B^\circ}{8}$$

$$\sin B^\circ = \frac{8\sin 35^\circ}{6} =$$

$$B^\circ = \sin^{-1}(.76)$$

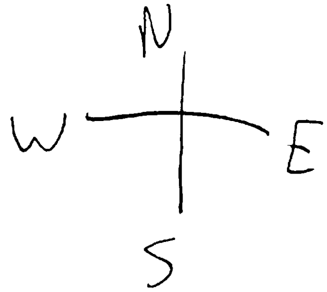
$$B^\circ = 49.9^\circ$$

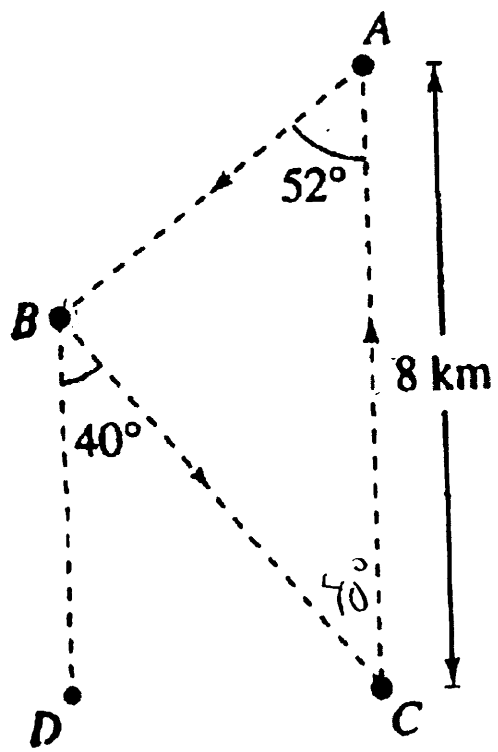
✓ $B_1 = 49.9^\circ$

✓ $B_2 = 130.1^\circ$

triangle 1		triangle 2
b=8	$B_1 = 49.9^\circ$	b = 8 $B_2 = 130.1^\circ$
a=6	$A = 35^\circ$	a = 6 $A = 35^\circ$
c=	$C = 95.1^\circ$	c = $C = 14.9^\circ$
c = 10.4		c = 2.7

Example 6 The course for a boat race starts at point A and proceeds in the direction S 52° W to point B, then in the direction S 40° E to point C, and finally back to A, as shown in figure. Point C lies 8 kilometers directly south of point A. Approximate the total distance of the race course.





$b = 8$ $\quad B = 88^\circ$

$a =$ $\quad A = 52^\circ$

$c =$ $\quad C = 40^\circ$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 88^\circ}{8} = \frac{\sin 52^\circ}{a}$$

$$\frac{a \sin 88^\circ}{\sin 88^\circ} = \frac{8 \sin 52^\circ}{\sin 88^\circ}$$

$a \approx 6.308$

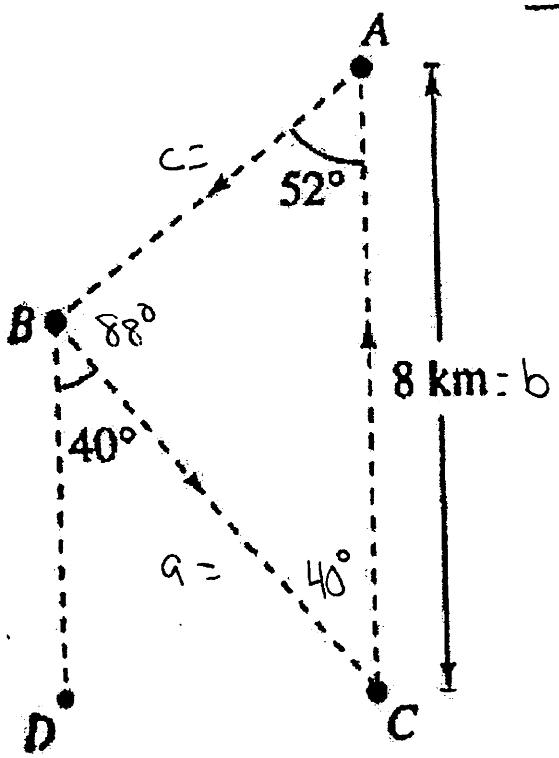
$$\frac{\sin 40^\circ}{c} = \frac{\sin 88^\circ}{8}$$

$$c = \frac{8 \sin 40^\circ}{\sin 88^\circ}$$

$= 5.145$

19.453

Example 7 The course for a boat race starts at point A and proceeds in the direction S 52° W to point B, then in the direction S 40° E to point C, and finally back to A, as shown in figure 6.9 (textbook page 415). Point C lies 8 kilometers directly south of point A. Approximate the total distance of the race course.



$B = 180° - 52° - 40° = 88°$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$= \frac{c}{\sin 40°} = \frac{8}{\sin 88°}$$

$$= c = \frac{8 \sin 40°}{\sin 88°} = \boxed{5.145}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 52°} = \frac{8}{\sin 88°}$$

$$a = \frac{8 \sin 52°}{\sin 88°} = \boxed{6.308}$$

add all sides

$$a + b + c = \boxed{19.453 \text{ km}}$$

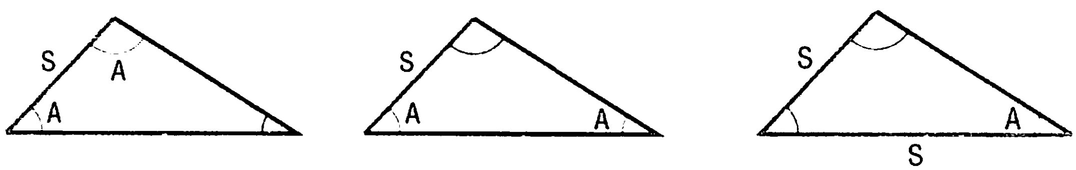
The **Law of Sines** is used to solve triangles for which Case 1 or 2 holds.

Theorem

Law of Sines

For a triangle with sides a, b, c and opposite angles A, B, C, respectively,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \tag{1}$$



Case 1: ASA

Case 1: SAA

Case 2: SSA

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \frac{\sin A}{a} = \frac{\sin C}{c} \qquad \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\boxed{A + B + C = 180°}$$