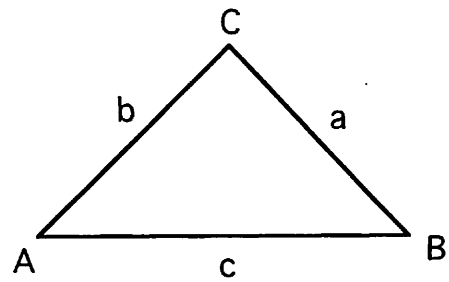
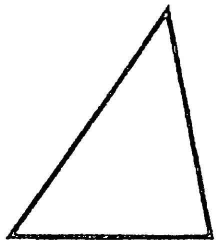
§ 8.3 Law of Cosines

The Law of Cosines is used to solve triangles in which two sides and the included angle (the angle between the two sides) are known or in which three sides are known
(SAS or SSS)

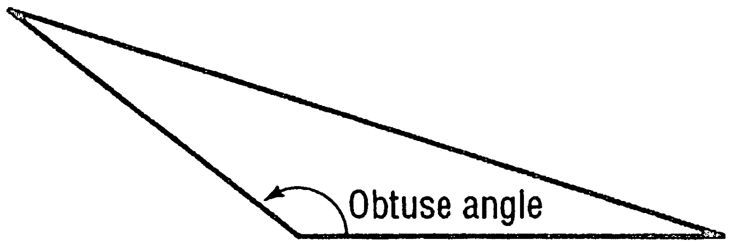


<u>The Law of Cosines</u> - If A, B and C are the measures of the angles of a triangle and a, b and c are the lengths of the sides opposite these angles, then
$a^2 = b^2 + c^2 - 2bc\cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos B$ or $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab\cos C$ or $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Note: It is wise to find the largest angle which is across the largest side FIRST !

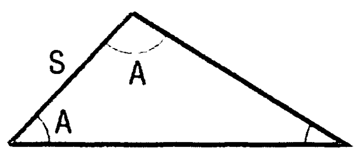


(a) All angles are acute

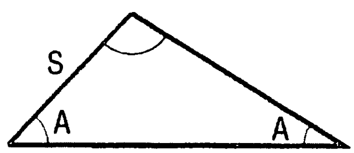


(b) Two acute angles and one obtuse angle

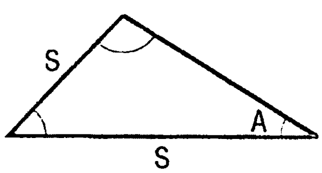
Case 1: One side and two angles are known (ASA or SAA).
Case 2: Two sides and the angle opposite one of them are known (SSA).
Case 3: Two sides and the included angle are known (SAS).
Case 4: Three sides are known (SSS).



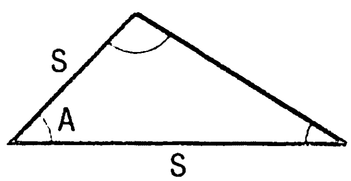
Case 1: ASA



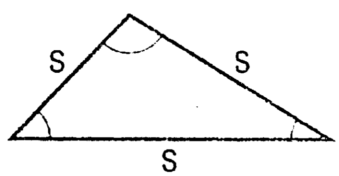
Case 1: SAA



Case 2: SSA



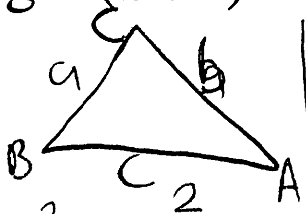
Case 3: SAS



Case 4: SSS

Example 1 Solve triangle (SSS)

a = 4, b = 3, and c = 6.



$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos C = \frac{4^2+3^2-6^2}{2(4)(3)}$$

$$\cos C = \frac{-11}{24} = \cos^{-1}\left(\frac{-11}{24}\right) = 117.3^\circ$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos B = \frac{16+36-9}{2(4)(6)}$$

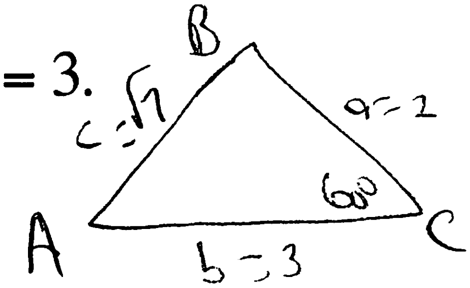
$$\cos B = \frac{43}{48}$$

$$\cos^{-1}\left(\frac{43}{48}\right) = 26.4^\circ$$

$$A = 180 - B - C = 180 - 117.3^\circ - 26.4^\circ = 36.3^\circ$$

Example 2 Solve triangle (SAS)

C = 60°, a = 2 and b = 3.



$$c^2 = a^2+b^2-2ab\cos C^\circ$$

$$c^2 = 2^2+3^2-2(2)(3)\cos 60^\circ$$

$$c^2 = 4+9-12\cos 60^\circ$$

$$c^2 = 13 - 12\left(\tfrac{1}{2}\right)$$

$$c^2 = 7$$

$$c = \sqrt{7} = 2.$$

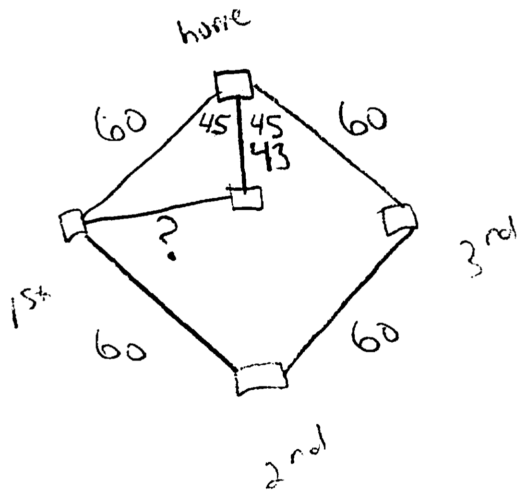
$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos B = \frac{2^2+(\sqrt{7})^2-3^2}{2(2)(\sqrt{7})}$$

$$\cos^{-1}\left(\frac{2}{4\sqrt{7}}\right) = 79.1^\circ$$

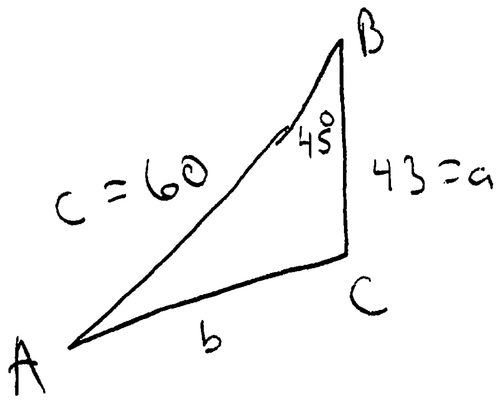
$$A = 180^\circ - 60^\circ - 79.1^\circ = 40.9^\circ$$

Example 3 The pitchers mound on a women's softball field is 43 feet from home plate and the distance between the bases is 60 feet. How far is the pitchers mound from first base?



SAS

~~SSS~~



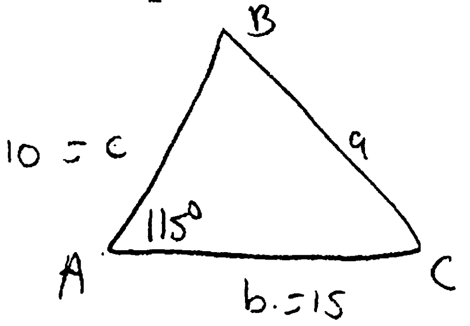
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = (43)^2 + (60)^2 - 2(43)(60)\cos 45^\circ$$

$$b^2 = 1800.3$$

$$b = 42.43 \text{ ft}$$

Example 2 Solve triangle (SAS) $A = 115°$, $c = 10$ and $b = 15$.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 15^2 + 10^2 - 2(15)(10)\cos 115°$$

$$a^2 = 451.785$$

$$\boxed{a = 21.26}$$

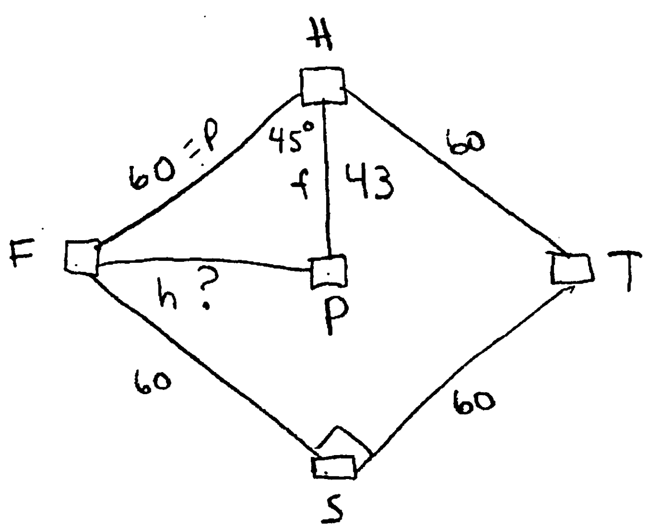
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$C = 180° - 115° - 39.75°$$

$$\sin B = \frac{b\sin A}{a} = \frac{15\sin 115}{21.26} = .6394 \qquad \boxed{C = 25.25°}$$

$$\sin B = .6394 \qquad \boxed{B = 39.75°}$$

Example 3 The pitchers mound on a womens softball field is 43 feet from home plate and the distance between the bases is 60 feet. How far is the pitchers mound from first base?



$\Delta HPF \quad H = 45°$

∞ SAS

$$h^2 = f^2 + p^2 - 2fp\cos H$$

$$h^2 = 43^2 + 60^2 - 2(43)(60)\cos 45°$$

$$h^2 = 1800.3$$

$$\boxed{h = \sqrt{1800.3} = 42.43 \text{ ft}}$$