Section 4.4 Properties of Definite Integrals

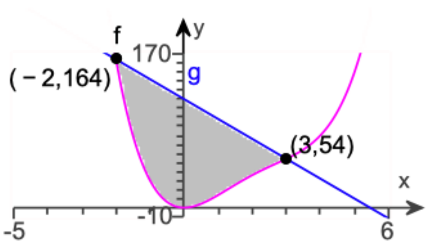
The additive property of definite integrals states that a definite integral can be expressed by as the sum of the two (or more) other definite integrals. If f is continuous over $[a, b]$ and we choose c such that $a < c < b$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

The area of a region bounded by the graphs of two functions, f and g, with $f(x) \geq g(x)$ over $[a, b]$ is

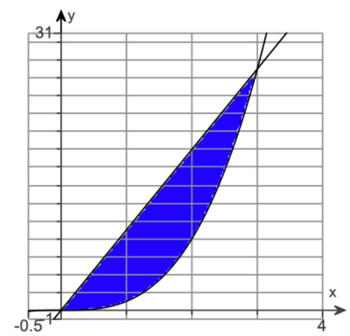
$$A = \int_a^b [f(x) - g(x)]\, dx$$

Example 1: Find the shaded area. SETUP only! Then pick one problem to work completely.

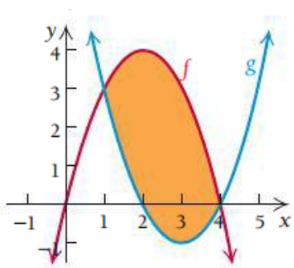
a) $f(x) = x^4 - 8x^3 + 21x^2$
$g(x) = -22x + 120$



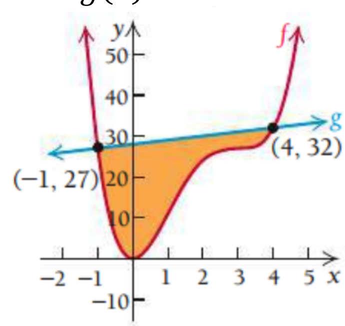
b) $f(x) = x^3$
$g(x) = 9x$



c) $f(x) = 4x - x^2$
$g(x) = x^2 - 6x + 8$

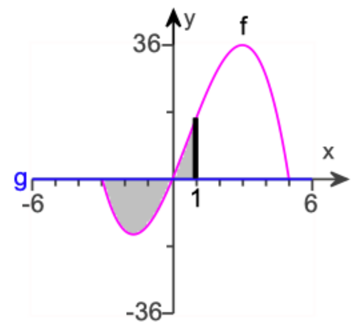


d) $f(x) = x^4 - 8x^3 + 18x^2$
$g(x) = x + 28$

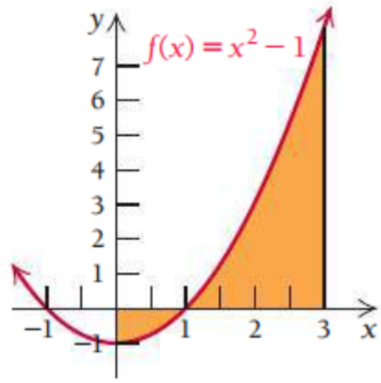


Example 2: Find the shaded area. SETUP only! Then pick one problem to work completely.

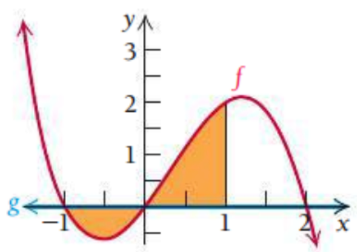
a) $f(x) = 15x + 2x^2 - x^3$
$g(x) = 0$



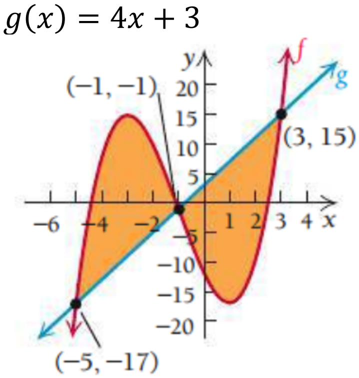
b) $f(x) = x^2 - 1$
$g(x) = 0$



c) $f(x) = 2x + x^2 - x^3$
$g(x) = 0$



d) $f(x) = x^3 + 3x^2 - 9x - 12$
$g(x) = 4x + 3$



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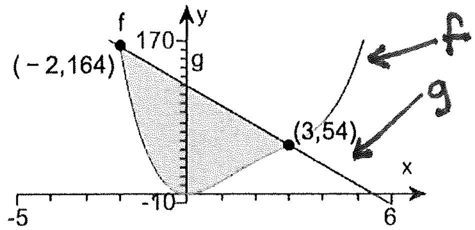
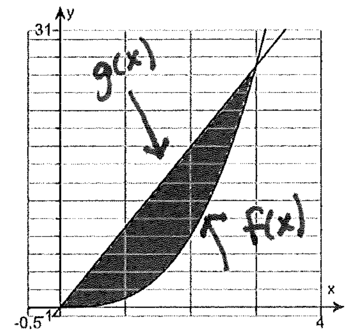


$$\int_{-2}^{3} (g(x) - f(x))\, dx$$

$= 375$

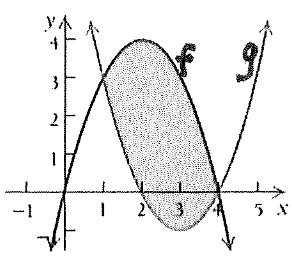
b) $f(x) = x^3$ curve
$g(x) = 9x$ line



$$\int_0^3 (g(x) - f(x))dx = \int_0^3 (9x - x^3)dx$$

$$= \frac{9}{2}x - \frac{1}{4}x^4 \Big|_0^3 = \frac{81}{4}$$

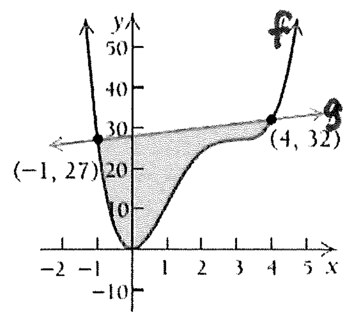
c) $f(x) = 4x - x^2$
$g(x) = x^2 - 6x + 8$



$$\int_1^4 (f(x) - g(x))\, dx$$

$= 9$

d) $f(x) = x^4 - 8x^3 + 18x^2$
$g(x) = x + 28$



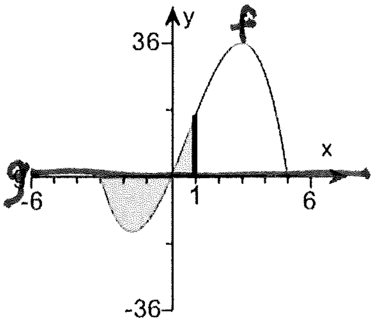
$$\int_{-1}^{4} (g(x) - f(x))\, dx$$

$= \frac{125}{2}$

Area is positive

Example 2: Find the shaded area. SETUP only! Then pick one problem to work completely.

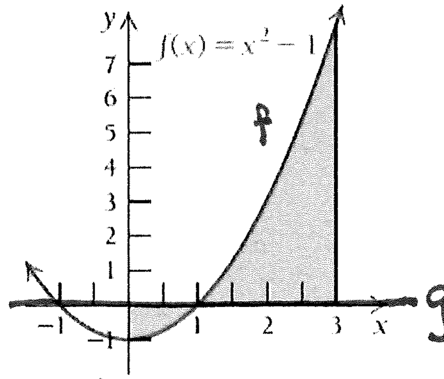
a) $f(x) = 15x + 2x^2 - x^3$
$g(x) = 0$



$$\int_{-3}^{0} (g(x) - f(x))\,dx + \int_{0}^{1} (f(x) - g(x))\,dx$$

$$= \frac{223}{6}$$

b) $f(x) = x^2 - 1$
$g(x) = 0$

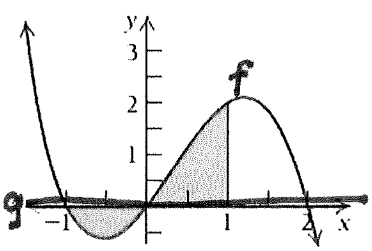


$$\int_{0}^{1} (g(x) - f(x))\,dx + \int_{1}^{3} (f(x) - g(x))\,dx$$

$$= \frac{22}{6}$$

(on next page)

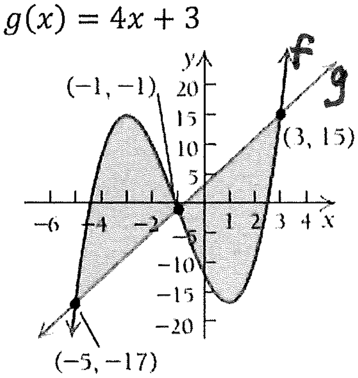
c) $f(x) = 2x + x^2 - x^3$
$g(x) = 0$



$$\int_{-1}^{0} (g(x) - f(x))\,dx + \int_{0}^{1} (f(x) - g(x))\,dx$$

$$= \frac{3}{2}$$

d) $f(x) = x^3 + 3x^2 - 9x - 12$
$g(x) = 4x + 3$



$$\int_{-5}^{-1} (f(x) - g(x))\,dx + \int_{-1}^{3} (g(x) - f(x))\,dx$$

$$= 128$$

Ex 2

b) $\int_0^1 g-f\,dx + \int_1^3 f-g\,dx$

$\int_0^1 0-(x^2-1)dx + \int_1^3 (x^2-1)-0\,dx$

$\int_0^1 -x^2+1\,dx + \int_1^3 x^2-1\,dx$

$\frac{-x^3}{3}+x \Big|_0^1 + \frac{x^3}{3}-x \Big|_1^3$

$\left(\frac{-1^3}{3}+1\right)-\left(\frac{-0^3}{3}+0\right) + \left(\frac{3^3}{3}-3\right)-\left(\frac{1^3}{3}-1\right) =$

$\left(-\tfrac{1}{3}+1\right)-(0) + (9-3)-\left(\frac{1}{3}-1\right) =$

$\frac{2}{3}+6-\left(-\frac{2}{3}\right) =$

$\frac{2}{3}+6+\frac{2}{3} =$

$\frac{22}{3}$